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Principles of Quantum Computation and Information

Volume I: Basic Concepts

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Volume I: Basic Concepts

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Printed in Singapore.

To Silvia
g.b.

To my wife for her love and encouragement
g.c.

To my family and friends
g.s.

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Preface

Purpose of the book

This book is addressed to undergraduate and graduate students in physics, mathematics and computer science. It is written at a level comprehensible to readers with the background of a student near to the end of an undergraduate course in one of the above three disciplines. Note that no prior knowledge either of quantum mechanics or of classical computation is required to follow this book. Indeed, the first two chapters are a simple introduction to classical computation and quantum mechanics. Our aim is that these chapters should provide the necessary background for an understanding of the subsequent chapters.

The book is divided into two volumes. In volume I, after providing the necessary background material in classical computation and quantum mechanics, we develop the basic principles and discuss the main results of quantum computation and information. Volume I would thus be suitable for a one-semester introductory course in quantum information and computation, for both undergraduate and graduate students. It is also our intention that volume I be useful as a general education for other readers who would like to learn the basic principles of quantum computation and information and who have the basic background in physics and mathematics acquired in undergraduate courses in physics, mathematics or computer science.

Volume II deals with various important aspects, both theoretical and experimental, of quantum computation and information. This volume necessarily contains parts that are more technical or specialized. For its understanding, a knowledge of the material discussed in the first volume is necessary.

General approach

Quantum computation and information is a new and rapidly developing field. It is therefore not easy to grasp the fundamental concepts and central results without having to face many technical details. Our purpose in this book is to provide the reader interested in this field with a useful and not overly heavy guide. Therefore, mathematical rigour is not our primary concern. Instead, we have tried to present a simple and systematic treatment, such that the reader might understand the material presented without the need for consulting other texts. Moreover, we have not tried to cover all aspects of the field, preferring to concentrate on the fundamental concepts. Nevertheless, the two volumes should prove useful as a reference guide to researchers just starting out in the field.

To fully familiarize oneself with the subject, it is important to practice solving problems. The book contains a large number of exercises (with solutions), which are an essential complement to the main text. In order to develop a solid understanding of the arguments dealt with here, it is indispensable that the student try to solve a large part of them.

Note to the reader

Some of the material presented is not necessary for understanding the rest of the book and may be omitted on a first reading. We have adopted two methods of highlighting such parts:

- 1) The sections or subsections with an asterisk before the title contain more advanced or complementary material. Such parts may be omitted without risk of encountering problems in reading the rest of the book.
- 2) Comments, notes or examples are printed in a small typeface.

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We are indebted to several colleagues for criticism and suggestions. In particular, we wish to thank Alberto Bertoni, Gabriel Carlo, Rosario Fazio, Bertrand Georgeot, Luigi Lugiato, Sandro Morasca, Simone Montangero, Massimo Palma, Saverio Pascazio, Nicoletta Sabadini, Marcos Saraceno, Stefano Serra Capizzano and Robert Walters, who read preliminary versions of the book. We are also grateful to Federico Canobbio and Sisi Chen. Special thanks is due to Philip Ratcliffe, for useful remarks and suggestions, which substantially improved our book. Obviously no responsibility should be attributed to any of the above regarding possible flaws that might remain, for which the authors alone are to blame.

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About the Cover

This acrostic is the famous *sator* formula. It can be translated as:

'Arepo the sower holds the wheels at work'

The text may be read in four different ways:

- (i) horizontally, from left to right (downward) and from right to left (upward);
- (ii) vertically, downward (left to right) and upward (right to left).

The resulting phrase is always the same.

It has been suggested that it might be a form of secret message.

This acrostic was unearthed during archeological excavation work at Pompeii, which was buried, as well known, by the eruption of Vesuvius in 79 A.D. The formula can be found throughout the Roman Empire, probably also spread by legionnaires. Moreover, it has been found in Mesopotamia, Egypt, Cappadocia, Britain and Hungary.

The *sator* acrostic may have a mystical significance and might have been used as a means for persecuted Christians to recognize each other (it can be rearranged into the form of a cross, with the opening words of the Lord's prayer, *A Paternoster O*, both vertically and horizontally, intersecting at the letter N, the Latin letters A and O corresponding to the Greek letters alpha and omega, beginning and end of all things).