Vibrational Mechanics

**Nonlinear Dynamic Effects,
General Approach, Applications**

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Nonlinear Dynamic Effects,
General Approach, Applications

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World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN 981-02-3890-8

Printed in Singapore by Uto-Print

Dedicated to the memory of Nina

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Preface to the Russian Edition

This book has two aims. The first aim is to describe a number of wonderful phenomena caused by the action of vibration on nonlinear mechanical systems. Here is a short list of such phenomena: vibrational displacement — the directed on the average “slow” motion or the change in the state of the system affected by “fast” undirected on the average mechanical actions (vibration); the change in the physical and mechanical properties of a body under vibration with regard to slow actions; the transformation of equilibrium states, in particular — their stabilization and destabilization under the action of vibration; the change in free oscillation frequencies of the system, caused by vibration; vibrational maintenance of rotation and self-synchronization of unbalanced rotors having an unexpected analogue in the peculiarities of the motion of celestial bodies; the seeming change in both magnitude and direction of the gravitational force. That enumeration is far from being complete. Many of those phenomena are widely used in technology and engineering. Still more effective use of such phenomena is expected in the nearest future. The author hopes this book will contribute to it.

The second, and perhaps the main aim of the book is to propose a general mechanical and mathematical approach to the description and investigation of this class of phenomena — the approach which may be called *vibrational mechanics*. Vibrational mechanics is a mechanics for the observer interested only in the “slow” motions of the system. Such motions appear together with the fast motions in a non-linear system under vibration and are, as a rule, of the main practical interest. It has been found that this observer, whom we call the observer **V** to distinguish him from the “ordinary” observer whom we will call the observer **O**, in order to make a correct description of the behaviour of the system must add to all the slow forces, acting upon the system, certain supplementary slow forces, called *vibrational forces*. These forces are calculated according to certain rules. Thus, the vibrational mechanics is in some sense analogous to the mechanics of a relative motion.

All the above-mentioned phenomena can be easily and naturally explained by the appearance of vibrational forces. On the other hand, the disregard of those forces leads to misunderstanding and errors, which happened in the past and are still happening now. We will discuss them in this book.

Vibrorheology is an important part of vibrational mechanics. It mainly studies the change caused by vibration in the rheological characteristics of bodies with regard to the slow actions, and it also studies the corresponding slow motions.

Vibrational mechanics and vibrorheology play an important role in the new section of the applied theory of oscillations which has been formed in recent years — the *theory of vibrational processes and vibrational devices*. This theory studies the regularities of excitation and effects of vibration in different mechanical systems. It also embraces the theory of the machines in which vibration is used for beneficial purposes.

A few more words about the positions of the observers **O** and **V**. - positions which are widely used in this book and in some illustrations are marked by symbols and . The letters **O** and **V** are the initial letters of the words “ordinary” and “vibrational”. In the world, perceived by the observer **O**, the ordinary laws of mechanics are valid in their direct form. Making and solving differential equations of motion, based on those laws, that observer gives correct descriptions of the phenomena, considered in the book. But his descriptions are rather intricate, with unnecessary details, as a result of which the interpretation of the results is sometimes quite embarrassing.

The observer **V**, as was mentioned, deliberately “does not notice” any fast forces or fast motions, though he does not forget that they do exist in reality. Staying within the frames of the “ordinary” mechanics, to make his descriptions correct, he must add to all the slow forces also the vibrational forces. The world of the observer **V** is much simpler than that of the observer **O**. In particular, the multidimensional system may be seen by him as a system of much smaller dimensions, an essentially non-conservative system may seem to him to be conservative, the discontinuous system may be seen by him as continuous or “smooth” and so on. As was mentioned, for the observer **V** vibrational mechanics and vibrorheology are mechanics and rheology respectively. In connection with the mistakes, which are often made when analyzing such phenomena, we can also speak of the position of the observer **W** (the first letter of the word “wrong”). He fails to take into account certain rather important circumstances. This observer either does not notice vibration and the fast forces, acting in the system, or does not take into consideration any possible consequences of their presence. So when considering the slow motions, he does not take into account a possibility of the appearance of vibrational forces. The world of this observer is full of “miracles”, riddles and paradoxes. Trying to explain them he sometimes begins to doubt the validity of the basic laws of mechanics — the law of conservation of energy, the law of action and reaction. He believes that under the action of vibration the weight of the body changes, that the velocity of the center of inertia of the system may change only at the expense of internal forces, etc.

However, the position of the observer $\mathbf{W}$ may sometimes also prove useful. In inventors' activities a temporal disregard of the laws of physics and mechanics, which hinder in attaining their aim, is an effective method of finding new technical solutions. We mean here the so called *fantastic analogy*, proposed by Gordon [176].

Now a few words about the terms *vibrational mechanics* and *vibrorheology*. Their introduction seems to be justified, first, by the peculiarity of the class of phenomena that are to be considered and, secondly, by the availability of a common methodical approach to the investigation of these effects. These terms, used by the author in his presentations and publications met no objections on the part of his colleagues [89, 92, 95, 100]. Equations, describing the slow motions, are called by us *the main equations of vibrational mechanics*. It is surprising and remarkable that for a comparatively wide class of systems these equations can be written in the form, typical of the potential system with dissipation, while the initial system is essentially non-potential. Systems of this type are named by us *potential on the average dynamic systems*. A number of important practical problems, in particular problems on synchronization (resonances) under the rotation of solid bodies, are reduced to studying those systems.

Vibrational mechanics is considered in this book as a special case of a more general concept which can be named *mechanics of systems with hidden motions*. Both in the process of mechanical-mathematical modeling real systems and in striving to simplify the models, one has a natural desire "not to see", "to partly ignore" certain components of motions of the system and even some degrees of freedom which seem to be of minor importance. The question arises: by which mechanics should the observer be guided lest he should come into conflict with reality? The answer is analogous to the previous one: certain additional forces should be introduced into the equations, describing the motions which are to be considered. Just like above, the mechanics of systems with partly ignored motions can be interpreted from the points of view of the three observers $\mathbf{O}'$, $\mathbf{V}'$ and $\mathbf{W}'$ similar to observers $\mathbf{O}$, $\mathbf{V}$ and $\mathbf{W}$. It is remarkable that vibrational mechanics is connected with both the classic works on mechanics and modern investigations. By the former we mean the works by Thomson, Tait, and Routh on mechanics of systems with cyclic coordinates, the works by Reynolds on the theory of turbulence, by Poincare and Lyapunov on celestial mechanics and theory of the stability of motion, by Krylov, Bogolyubov, Mitropolsky and Malkin on methods of averaging and the theory of periodic solutions of differential equations with a small parameter. By the latter we mean the works by Kapitsa and his followers on the behavior of a pendulum with a vibrating

axis of suspension, on the theory of vibrational processes and devices and also on the problem of resonances in the Solar system. That common character of ideas and their continuity has been systematically reflected in this book.

The first part of the book is devoted to the theoretical basis of vibrational mechanics, in particular to the description of the general approach, mentioned above, which is mainly based on the so called *method of direct separation of motions*. The main statements have been formulated here as theorems. It should be noted however that the elaboration of the mathematical apparatus of vibrational mechanics cannot as yet be considered completed. Other chapters of the book are devoted to numerous applications of vibrational mechanics.

The book is provided with a detailed table of contents and detailed captions to figures. It seems to us both of them will be helpful to the readers.

The book is meant first of all for the specialists in both theoretical and applied mechanics, in the theory of non-linear oscillations and vibrational technology. It may also prove to be useful for mathematicians involved in the theory of ordinary differential equations as a source of some new problems.

While working at the concept and apparatus of vibrational mechanics I had a happy chance to discuss the main points with V.I.Babitsky, V.V.Beletsky, V.V.Koslov, M.Z. Kolovsky, P.S.Landa, L.G.Loytsyansky, Ja.G.Panovko, A.A.Pervosvansky, V.V.Rumyantsev, O.V.Savinov, G.Ju.Stepanov, and K.V.Frolov. My young colleagues E.B.Kremer, O.Z.Malakhova, A.V.Petchenev, A.Ya. Fidline and N.P.Yaroshevich both on my request and on their own initiative made a number of complicated researches the results of which have been reflected in the book. Besides, O.Z.Malakhova read the manuscript very attentively and helped to eliminate some drawbacks. While reviewing the book, important remarks were made by M.Z.Kolovsky. I express my deep gratitude to those scientists.

I appreciate the unfailing understanding and support of my colleagues and the management of the Research and Design Institute of Mechanical Treatment of Mineral Resources ("Mekhanobr", St-Petersburg) which made it possible for me to devote myself to this most interesting work.

I am greatly thankful to my wife Nina Granat who passed away when the work on the book was practically completed. A gifted researcher herself (the reader will find some results of her work in the book), Nina selflessly released me from any domestic care. To Nina's fond memory this book is dedicated.

November 29, 1991, St.Petersburg

I.Blekhman

Preface to the English Edition

I am thankful to the World Scientific Publishing Co. for the opportunity to present my book to the English readers.

Five years have passed since the publication of this book in Russia [125]. The ideas and results stated there have been essentially developed and supplemented during that time. In this English edition of the book the author tended to at least partially reflect those developments:

- item 3.3 has been added, which is devoted to the characteristics of the advantages and limitations of the approach and methods stated in the book;
- item 5.3 has been added, which gives a short review of the results referring to the stability of composite pendulums with a vibrating axis of suspension, including the follower-loaded ones;
- item 7.8 was supplemented by the general definition of synchronization;
- item 10.6, devoted to the vibrational pumps and coaches has been considerably enlarged;

a new chapter had been introduced. This chapter (chapter 18) contains some ideas on the problem of controlling the properties of the nonlinear systems by means of vibration. Here the new approach to the creation of the materials with given dynamic properties — the so called dynamic materials — is discussed most thoroughly;

- the bibliography contains additional references to both the latest investigations and to those made before, which unfortunately escaped the attention of the author when the book was being prepared.

In 1996 the author delivered a PhD course of lectures on vibrational mechanics at the Department of Solid Mechanics, headed by Professor Pauli Pederson, at the Technical University of Denmark (DTU).

Then similar lectures were delivered at the Institute of the Problems of Mechanical Engineering of the Russian Academy of Sciences; at the St. Petersburg Technical University (the former Polytechnical Institute) and at the Marine Technical University.

Professors D. A. Indeitsev, M. Z. Kolovsky and S. V. Sorokin, who had invited me, and their disciples as well as their colleagues at the DTU made an attentive and interested audience. The contact with them proved to be

both pleasant and useful. Some of them, including Professors J. J. Thomsen, S. V. Sorokin and their pupils J. S. Jensen, D. M. Tchernyak and E. C. Miranda found it interesting to make their own investigations of that problem. References to their investigations and short reviews of them are given in the book.

The author's contacts with Professors A. L. Fradkov, K. A. Lurie, and L. Sperling were both very useful and very pleasant.

I am deeply grateful to all those scientists.

I want to thank Associate Professor Minna Perelman for her careful translation in the process of which some fragments of the text were improved.

I am also thankful to my colleague Larissa Titova for her valuable assistance in preparing the manuscript.

I am grateful to Professor Mikhail Levinshtein for his helpful advice.

July 30, 1999, St.Petersburg

I.Blekhman

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