INVERSE PROBLEMS FOR ELECTRICAL NETWORKS

SERIES ON APPLIED MATHEMATICS

Editor-in-Chief: Frank Hwang

Associate Editors-in-Chief: Zhong-ci Shi and U Rothblum

- Vol. 1** International Conference on Scientific Computation
eds. T. Chan and Z.-C. Shi
- Vol. 2** Network Optimization Problems — Algorithms, Applications and Complexity
eds. D.-Z. Du and P. M. Pandalos
- Vol. 3** Combinatorial Group Testing and Its Applications
by D.-Z. Du and F. K. Hwang
- Vol. 4** Computation of Differential Equations and Dynamical Systems
eds. K. Feng and Z.-C. Shi
- Vol. 5** Numerical Mathematics
eds. Z.-C. Shi and T. Ushijima
- Vol. 6** Machine Proofs in Geometry
by S.-C. Chou, X.-S. Gao and J.-Z. Zhang
- Vol. 7** The Splitting Extrapolation Method
by C. B. Liem, T. Lü and T. M. Shih
- Vol. 8** Quaternary Codes
by Z.-X. Wan
- Vol. 9** Finite Element Methods for Integrodifferential Equations
by C. M. Chen and T. M. Shih
- Vol. 10** Statistical Quality Control — A Loss Minimization Approach
by D. Trietsch
- Vol. 11** The Mathematical Theory of Nonblocking Switching Networks
by F. K. Hwang
- Vol. 12** Combinatorial Group Testing and Its Applications (2nd Edition)
by D.-Z. Du and F. K. Hwang
- Vol. 13** Inverse Problems for Electrical Networks
by E. B. Curtis and J. A. Morrow

Series on
Applied Mathematics
Volume 13

INVERSE PROBLEMS FOR ELECTRICAL NETWORKS

Edward B. Curtis
James A. Morrow

Mathematics Department
University of Washington, Seattle
USA

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Curtis, Edward B., 1933–

Inverse problems for electrical networks / Edward B. Curtis and James A. Morrow.

p. cm. (Series on applied mathematics – Vol. 13)

Includes bibliographical references and index.

ISBN 9810241747 (alk. paper)

1. Electric networks. 2. Inverse problems (Differential equations) 3. Matrices. I.

Morrow, James A., 1941– II. Title. III. Series.

TK454.2.C87 2000

621.3'01'51535--dc21

99-053875

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 2000 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

Printed in Singapore by Uto-Print

Preface

This book is the result of an accumulation of work done by the authors and their students over the past twelve years. In each of the years listed, 8-10 students were brought to the University of Washington for a summer REU (Research Experience for Undergraduates) program, supported by a REU Grant from the NSF. We want to thank the NSF for its support during this period. And we want to thank the students for their enthusiasm, their dedication, and their individual contributions, without which this book would not have been possible.

REU Students 1988

Sara A. Beavers, Thaddeus J. Edens, Jeffrey E. Eldridge, Troy B. Holly, Christina H. Lamont, Olga M. Simek, Laura A. Smithies, Hsi-Jung Wu, Matthew J. Curland.

1989

Michael C. Carini, Robert A. Coury, Richard P. Dechance, Peter L. Engrav, Matthew G. Hudelson, Jeannie C. Mah, John Morgan Oslake, Michael J. Parks.

1990

Eric J. Auld, John T. Guthrie, Joshua K. Landrum, Adrian V. Mariano, Edith A. Mooers, Miriam A. Myjak, Brett A. Sovereign, Peter Staab, Stefan G. Treatman.

1991

Michael W. Buksas, Jim L Carr, Peter B. Gilbert, Benjamin Thaddeus Kosnik, Victor Lee, Nancy McNally, Michael J. Mills, Jami M Moksness.

1992

David Dorrough, Kristine Fromm, David Ingerman, Kurt Krenz, Keli Kringle, Justin Mauger, Julie Olsen, Kevin Rosema, Konrad Schroder, James Warren.

1993

Christopher Cook, Andrew Iglesias, Laura Judd, Matthew Munro, Aleksandr Murkes, David Muresan, Chris Higginson, Konrad Schroder, Neil York, Leonid Zheleznyak.

1994

Nathan E. Bramall, Sean P. DeMerchant, Darin Diachin, James A. Herzog, Todd Hollenbeck, Keith Johnson, Michael McLendon, Erika L. Schubert, Tung T. Tran, David Vanderweide.

1996

Margaret Chaffee, Amy Ehrlich, Mark Hoefer, Derek Jerina, Dmitriy Leykekhman, Phillip Lynch, Marc Pickett, Aubin Whitley.

1997

James Bisgard, Benjamin Blander, Ryan Daileida, Darren Lo, Sreekar M. Shastry, Spencer Shepard, Chris Staskewicz, Ryan Yamachika.

1998

Tarn Adams, Neil Burrell, Laura Kang, Kjell Konis, Jeffrey Mermin,

Amanda Mueller, Laura Negrin, Derek Newland, Julie Rowlett, Ryan Sturgell.

1999

Ingrid Abendroth, Thomas Carlson, Rod Huston, Carla Pellicano, Chris Romero, Mike Usher, John Thacker, Christopher Twigg.

This page is intentionally left blank

Contents

Preface	v
1 Introduction	1
1.1 Electrical Networks	1
1.2 Other Topics	9
2 Circular Planar Graphs	11
2.1 Connections	11
2.2 $Y - \Delta$ Transformations	14
2.3 Edge Removal	16
2.4 Trivial Modifications	18
2.5 Well-connected Graphs	20
3 Resistor Networks	27
3.1 Conductivities on Graphs	27
3.2 The Response Matrix	32
3.3 The Kirchhoff Matrix	33
3.4 The Dirichlet Norm	35
3.5 The Schur Complement	40
3.6 Sub-matrices of the Response Matrix	47
3.7 Connections and Determinants	49
3.8 Recovery of Conductances I	55
4 Harmonic Functions	59
4.1 Harmonic Continuation	59
4.2 Recovering Conductances from Λ	62
4.3 Special Functions on Networks	67

4.4	Special Functions on G_{4m+3}	71
4.5	Recovery of Conductances II	74
4.6	The Differential of L	77
5	Characterization I	83
5.1	Properties of Response Matrices	83
5.2	Some Matrix Algebra	85
5.3	Parametrizing Response Matrices	86
5.4	Principal Flow Paths	90
5.5	Proof of Theorem 5.1	93
6	Adjoining Edges	99
6.1	Adjoining a Boundary Edge	99
6.2	Adjoining a Boundary Pendant	103
6.3	Adjoining a Boundary Spike	105
6.4	Recovery of Conductances III	108
7	Characterization II	109
7.1	Totally Non-negative Matrices	109
7.2	Characterization of Response Matrices II	117
8	Medial Graphs	121
8.1	Constructing the Medial Graph	121
8.2	Coloring the Regions	124
8.3	Switching Arcs	126
8.4	Lenses	127
8.5	Uncrossing Arcs	130
8.6	Families of Chords	134
8.7	Standard Arrangements	140
9	Recovering a Graph	149
9.1	Connections	149
9.2	The Cut-point Lemma	152
9.3	Recovering a Medial Graph	158
9.4	Examples	159
9.5	Critical Graphs	165
10	Layered Networks	173

CONTENTS

xi

References

181

Index

183