FOUNDATIONS OF QUANTUM Third Edition CHROMODYNAMICS

An Introduction to Perturbative Methods
in Gauge Theories

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To my friends,

W.A.B., A.J.B. and D.W.D.

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An Introduction to Perturbative Methods
in Gauge Theories

T Muta

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FOUNDATIONS OF QUANTUM CHROMODYNAMICS (3RD EDITION)

An Introduction to Perturbative Methods in Gauge Theories

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PREFACE TO THE THIRD EDITION

In ancient Chinese literature, it is said that time passes as if it were a galloping white horse crossing in front peered through a narrow gap. My white horse runs fast as well. Already more than 20 years have passed since the first edition of this book was published. Graduate students at that time who studied Quantum Chromodynamics (QCD) with this book are now in their late forties. Many of them may now be professors in particle physics who would be using this book for their students. I would be extremely happy if the book has helped all those people to get a deeper understanding of the basis of QCD.

In this third edition, I tried to keep the original form of the book as far as possible and so the changes are mostly minor corrections of misprints found after the second edition was published. I also included in this edition some memorable pictures that I took a long time ago.

I would like to thank Tomohiro Inagaki, who is an associate professor in Hiroshima University and was a graduate student when the first edition was released, for his full support in the whole process of preparing the third edition. The cover picture (galaxy M74) was selected from many pictures of celestial objects taken by Koji S. Kawabata at the Higashi-Hiroshima Observatory in Hiroshima University whom I would like to acknowledge. The Observatory was established when I was serving as President of Hiroshima University. I would like to thank Hiromi Sugihara for her continuous help in the preparation of the third edition and also with my work as President of Fukuyama University. I am particularly grateful to John Albert Kearley, Jr. who gave me useful advices on how to improve my English sentences.

T. Muta

*Hiroshima, Japan
April 2009*

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PREFACE TO THE SECOND EDITION

It is to my surprise that the basis of quantum chromodynamics (QCD) has been made firmer and firmer in its applicability in the perturbative as well as nonperturbative regime since the first edition of this book was prepared in 1986. While much theoretical progress has been made in perturbative QCD, the basic structure of its formulation has remained unchanged and so the book is still of value for researchers who wish to learn fundamental items of perturbative QCD.

In preparing the second edition, I thoroughly inspected the book to make corrections of misprints as many as possible. Two graduate students, Kazutaka Kawakami and Norikazu Yamada, were of great help in this tiring and time-consuming work. I would like to thank them for their beautiful work. In the second edition, some new sentences are added corresponding to the new developments in theoretical and experimental studies of QCD. Advices given to the author by Takashi Ohsugi, Yoshio Sumi and Reisaburo Tanaka have been very useful in the process of renewing the book. The appendix D is added for the convenience of the readers who are not used to employing the Feynman rule in our book but accustomed to the traditional Feynman rule which appears in many books.

T. Muta

*Hiroshima, Japan
March 1997*

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PREFACE TO THE FIRST EDITION

The strong interactions of fundamental particles have been successfully described by a non-Abelian gauge field theory called quantum chromodynamics (QCD). Because of its outstanding property of asymptotic freedom, a perturbative treatment makes sense for short-distance phenomena and predictions of the theory have been tested in a variety of elementary particle reactions. On the other hand, it is most important to control the nonperturbative regime of QCD in order to fully understand the strong interactions of hadrons. Unfortunately, however, no established method has surfaced to deal with the nonperturbative regime of QCD although some promising attempts are available.

In this book the techniques developed for the perturbative regime of QCD are explained in detail. To do this I have attempted to start at a level as low as that of a first-year graduate student's and continue from there to discuss the subject in a self-contained manner as far as possible. As a consequence the part of the book on background material for QCD became rather bulky. I think this part may also serve as a comprehensive introduction to gauge field theories. In the rest of the book the renormalization group method, the operator-product expansion and other technical developments are elucidated, and QCD predictions based on these techniques are derived for short-distance reactions. In the final chapter the infrared divergence is discussed in some detail since it is of relevance to the argument on the validity of the perturbative treatment.

The book grew out of lectures given in many places. I would like to thank those people who participated in the lectures and who made valuable criticisms.

During the course of writing the manuscript I have benefited from discussions and correspondences with many people as well as their assistance. I am happy to express my thanks to my friends, Bill Bardeen, Andrzej Buras and Dennis Duke for many discussions and their encouragement. Discussions with Kazuo Fujikawa, Yoichi Kazama, Noboru Nakanishi and Akio

Sugamoto have been very useful and I wish to express my gratitude to them. I am indebted to Tony Zee for valuable correspondences and to Hisao Nakkagawa and Akira Niégawa for critical comments. I would like to express my gratitude to Kiyoshi Kato, Tomoo Munehisa, Tatsumasa Tsurugai, and Sakue Yamada for providing me with experimental data and Monte Carlo results used in the book. The criticisms and comments by my colleagues and students have been undoubtedly beneficial. My hearty thanks are due to Jiro Kodaira, Toshinobu Maehara, Junji Okada, Juichi Saito, Ryuichi Najima, Masato Inoue, Kenji Hamada and other members of our laboratory.

The idea of turning my lecture notes into a book was first suggested to me four years ago by Prof. K.K. Phua. Since then he has waited for the completion of the book with great patience and kindness for which I would like to express my sincere gratitude. I would also like to thank Yasuo Hara for his concern about the completion of my book and his constant encouragement.

My friend and colleague, Yoshio Sumi, deserves many thanks for his kind help in drawing all the figures in the book in spite of his busy teaching and research schedule. It is my pleasure to thank my colleague, Masayoshi Kikugawa, for his help in computerizing the typing procedure, and Minoru Yonezawa and Seiichi Wakaizumi for their encouragement and support while I was writing the manuscript. A note of gratitude is due also to Hiroko Nishiura for her invaluable help in typing and correcting the whole manuscript. Without her persevering assistance the present book would not have materialized. I would also like to thank Emi Nakamoto for her cheerful assistance in typing the manuscript. Finally, I wish to thank my mother for her constant support of my research activities and my family for their patience during the years of preparing the manuscript.

Hiroshima, Japan
December 1986

T. Muta

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D. J. Gross at 1978 International Conference on High Energy Physics in Tokyo

H. D. Politzer at 1978 International Conference on High Energy Physics in Tokyo

From left to right, W. A. Bardeen, A. J. Buras, D. W. Duke and T. Muta in 2005 at Fermilab