NONLINEAR NONINTEGER ORDER CIRCUITS AND SYSTEMS—An Introduction

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— AN INTRODUCTION**

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Preface

Why this book?

Since 1993 our group (System and Control Group, Università degli Studi di Catania) has been involved in the study of non integer order systems exploring, in particular, specific topics related to the general subject. Recently in the literature more and more papers have been published on the theme, showing a renewed interest in the area by various researchers. Stimulating applications required by industrial companies have also reinforced the initial interest.

The main body of the book condenses some of the work done in the last years by our research group including also the basic principles of non integer order systems. The topic of non integer order “systems and circuits” is a fascinating one. In this monograph the basic principles of non integer order systems are integrated with the concepts of chaos, complexity and nonlinear cellular networks.

The main aim of the work is not to deal in detail with the theoretical and historical aspects of the well studied subject of non integer order systems. The book mainly regards the engineering aspect of the use of non integer order systems, with particular emphasis on the role that their use could play in conceiving new circuits for the study and use of complex systems. Our main purpose is to present in a clear and concise way a set of ideas regarding the non integer systems and their applications together with some tools for their study.

The book is intended for graduate students and researchers involved in the research area of nonlinear science and its real world application. The demonstration of the usefulness in practice of the non integer order systems is the guide followed in the book.

The presentation has been organized in such a way to remark the interdisciplinary character of the various topics and the applicability of the non integer order systems in the area of circuits and nonlinear network array design.

The structure of the book is the following. The first two chapters deal with the basic principles regarding the non integer order systems, with

particular emphasis on linear systems. The third chapter is focused on a special topic regarding the approximation problem of non integer order systems and its dual problem of the use of non integer order systems to manage high order integer systems. The fourth chapter introduces the role of the considered systems for control aims. The linear case introduces the nonlinear one where new results regarding chaos control are discussed and commented. The following two chapters respectively regard the non integer order CNN and the fundamental role that the non integer order systems play in the generation of a new class of circuit with strange behavior. The improvements of non integer order CNN with respect to the integer order ones are discussed and some examples are reported.

In the sixth chapter, some experimental circuits will be analyzed after having proved the existence of chaos by using analytical approaches. In fact, some design procedures will be introduced to design non integer order systems with chaotic behavior. In particular, the concept of non integer order cells is addressed with a view to considering complex system design.

In the last chapter some concluding remarks are reported. The actual status of the dealt subject and the future trends of the research in the area of non integer order systems are summarized.

The Appendix includes the Matlab and C codes of the tools used in the development of the various examples reported in the book.

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Catania, Italy, June 2000