A Primer on the Physics of the Cosmic Microwave Background

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A Primer on the Physics of the Cosmic Microwave Background

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To Sergio Fubini

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Preface

More than three score years ago, high-energy physicists were driven to scrutinize the properties of the cosmic radiation then available (i.e. cosmic rays). Today the same situation is realized not only with cosmic rays but also with different cosmological data: most notably, with the Cosmic Microwave Background (CMB in what follows). While I am writing this preface, European science is at the forefront of the developments in high-energy physics and cosmology thanks to the Large Hadron Collider program and thanks also to the Planck explorer mission. Today laboratory physics and celestial physics give us contradictory indications: it seems that all the matter accessible to terrestrial laboratory experiments contributes only 5% to the total energy budget of the Universe.

Cosmologists and astrophysicists today cannot ignore the knowledge of the micro-world provided by high-energy physics. In similar terms, high-energy physicists cannot avoid being exposed to some of the key concepts in modern gravitation and cosmology. While grand unifications of all fundamental forces are one of the intriguing hopes suggested by current theoretical speculations, the opportunity of a small unification lies already before us in the years to come: the construction of a common language which will allow, in the near future, a more effective exchange of information and ideas between contiguous branches of the physics community. The present book seeks to be a modest contribution to this mighty endeavor.

This book grew through the last decade because of various series of lectures that were either directly or indirectly connected to CMB physics and, more generally, to gravitation. In the last couple of years I came to the conclusion that an effective way of presenting a cosmology course (either for last year undergraduate or for PhD students) is to use CMB as a guiding theme. While lecturing to PhD students I have been confronted with the

problem of giving a sufficiently accurate and updated information to an audience that was, very often, rather composite. Not all PhD students were exposed to General Relativity or field theory in their undergraduate courses. Similarly, not all PhD students have a preliminary knowledge of astrophysics. I have tried, therefore, to present the material in a reasonably self-contained manner also in view of the time limitations imposed by a PhD course.

My warm acknowledgment goes to the Enrico Fermi center for a senior research grant entitled *From the Planck Scale to the Hubble Radius*. Without this support my efforts would have been forlorn. In commencing this script I wish also to express my very special and sincere gratitude to Prof. G. Cocconi and Prof. E. Picasso. I am indebted to G. Cocconi for his advice in the preparation of the first section. I am indebted to E. Picasso for delightful discussions which have been extremely relevant both for the selection of topics and for the overall quality of the manuscript.

Massimo Giovannini

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