A GALLERY OF CHUA ATTRACTORS

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A GALLERY OF CHUA ATTRACTORS

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*To my beloved family
Piero, Francesca and Arrigo*

*To my charming daughters
Maria Fiorella and Eleonora*

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PREFACE

Chua's circuit — the central theme of this book — is one of the most important scientific discoveries of recent times and one of the most fertile fields of research in complexity science. The book comes twenty-five years after Chua's original work, and fifteen after Madan's celebration of the circuit as a “*paradigm of complexity*”. Without attempting to provide a complete picture of research in the field, it provides an up-to-date review of the relevant literature and a “gallery” of nearly a thousand chaotic attractors generated by the circuit and its generalizations.

The book brings together six tutorial review papers, originally published in 2007, by the *International Journal of Bifurcation and Chaos*:

1. Bilotta, E., Pantano, P. & Stranges, S. [2007a] “A gallery of Chua attractors. Part I,” Vol. **17**, 1–60.
2. Bilotta, E., Pantano, P. & Stranges, S. [2007b] “A gallery of Chua attractors. Part II,” Vol. **17**, 293–380.
3. Bilotta, E., Pantano, P. & Stranges, S. [2007c] “A gallery of Chua attractors. Part III,” Vol. **17**, 657–734.
4. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007d] “A gallery of Chua attractors. Part IV,” Vol. **17**, 1017–1078.
5. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007e] “A gallery of Chua attractors. Part V,” Vol. **17**, 1383–1511.
6. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007f] “A gallery of Chua attractors. Part VI,” Vol. **17**, 1801–1910.

For this edition, we have rechecked the original papers and removed misprints. Section and equation numbers remain unchanged. All references to sections or equations contain a reference to the chapter in which they appear.

The main text is preceded by a broad-ranging “Prologue”. This provides detailed “recipes” for readers who wish to build Chua's circuit for themselves, links to software simulating the circuit with dimensionless equations, and a description of how it can be used to produce music. This last part uses findings originally reported in a review-tutorial of 2005 and further elaborated in two lectures at Berkeley, in March 2007. For the original paper see:

Bilotta, E., Gervasi, S. & Pantano, P. [2005] “Reading complexity in Chua's oscillator through music. Part I: A new way of understanding chaos,” *Int. J. Bifurcation and Chaos* **15**, 253–382.

Finally we present covers from the six issues of the *International Journal of Bifurcation and Chaos* in which our work originally appeared and the covers of two additional issues of the journal, also based on our work.

For additional references to the multimedia materials and software referred to in the book please visit:
<http://galileo.cincom.unical.it/chua>

The inside covers show pictures of attractors generated by the Chua circuit and are based on a poster originally created in 2005. It was this poster that inspired us to create our Gallery.

Together with the book, readers will find an audio CD with thirteen pieces, obtained by editing the original music generated by Chua's circuit. The first ten were obtained by applying musical styles to the sequences produced by the circuit. The last two are closer to the originals; here the only edits are minor changes designed to enhance the differences with respect to the earlier pieces and to bring out the expressive power of the music. In what follows, we provide additional details on the edited pieces.

Track 1

This piece was produced from a selection of tunes produced by the Chua attractor. The tunes are organized to provide both a basic structure for the piece and improvised solos. The overall form resembles "fusion" music.

Track 2

Here a single time series is subjected to a sequence of transformations, each of which doubles the previous tempo. The same sequences of notes are transposed into different keys and composed into a longer sequence — creating the arpeggio in the cello part. Different notes in the time series are played by different instruments. In the second part of the piece, the original time series is filtered and a delay effect applied.

Track 3

This piece explores the possibility of creating traditional musical forms, using material produced by the Chua attractor. The piece is based on selected melody fragments, transposed into different keys with different tempos. The voices are organized in the form of a canon.

Track 4

This piece is based on two different time series representing similar melodies which have both been produced in the same way. The first can be recognized by the initial beat and the arpeggio on the harp, the second from the sound of the oboe and the violins. Parts of the piece are transposed, creating harmonic passages and modulations into other keys.

Track 5

Here, the three coordinates of the Chua attractor are used to code the pitch, volume and duration of the notes. This raw material is then used as inspiration for the final arrangement. As previously, the transformations applied to the raw material produce arpeggios which are used both in the accompanying and the solo parts. Sometimes we can hear the melody modulated from the minor to the major. The way the Chua attractor controls the dynamics, gives the piece a natural, lively feel.

Track 6

This piece attempts to recreate classical feelings of "lightness". A single time series is subjected to various time transformations, with each version being played on a different instrument. Thus the slowest version is played on the double bass; the harp plays an arpeggio — a transposition of the original melody — at an intermediate tempo. The original melody — without modification — is played by the violins. Individual musical phrases are brought out by selecting specific notes from the time series and deleting others.

Track 7

The raw material for this piece was produced by filtering the data from the attractor to produce scales in different keys. The filtering was performed in real time, allowing the composer to perceive passages in the music as they emerged and to control the volume. This raw material was then subjected to a selection procedure to identify the most interesting parts of the work. Finally, a completely new bass line was added, giving meaning to the music.

Track 8

Different parts were obtained by transforming note durations in a single original time series. For example, the synthesizer we hear in the background, plays its notes at twice the speed of the notes played by the bass. The guitar plays two times faster than the synthesizer. The second part of the piece is marked by the return of the original time series, played on a second synthesizer which applies a series of sound effects.

Track 9

In this piece, the aim was to create musical structures similar to those we find in serious music from the beginning of the XX century. What is special here, is the way the time series are used to control the percussion instruments. In the orchestration, the different coordinates are used to control different families of instruments: (X: Woodwinds, Y: Percussion, Z: Strings).

Track 10

This piece uses three distinct time series to create a complex orchestrated structure.

Whenever the values lie within a specific range they are assigned to specific instruments. This creates the impression of a musical dialog. The durations of the notes are subjected to a series of transformations, creating a contrast between moments of relative ecstasy and calm.

Track 11

This piece presents the chaotic expressiveness of a Chua attractor, without any arrangement required by the user.

The three time-series of the system play three different musical instruments. The tempo of the notes is organized using a rhythmic sampling process, that is able to create melodic themes and variations. Moreover, the values provided from the three time-series are sampled with different durations in order to make the generated melodic sequences more recognizable.

It is possible to notice some perceptive structures due to the chaotic system. These structures change over time even if the main feeling seems repetitive, since the shape of the attractor remains fixed. The musification process uses a chromatic scale.

Track 12

This fragment shows the expressive capabilities of the circuit without any external arrangement. Sample times are very narrow, causing a perceptive cohesion and the creation of new timbres. The formation of these phenomena are possible since there are many repeated melodic expressions that capture the listener's attention on single frequency or groups of frequencies.

The auditory representation allows the listener to grasp the dynamic nature of Chua's attractor, which we have translated into music. Three different instruments follow the evolution of the note values, sometimes giving us organized melodic patterns, and sometimes chaotic ones.

Track 13

This is a collection of many pieces obtained from different Chua attractors and using different codification processes. In each composition, the three time-series of the system play three different musical instruments.

Some compositions are realized using an instrument to provide melodic sequences, while others use the time-series to produce varying timbres. The tempo of the notes is organized by using a rhythmic sampling process, that is able to create some melodic and non-melodic themes and variations.

We wish to thank Enrico Cupellini and Costantino Rizzuti who took our original material and created the undoubtedly suggestive pieces we present here. We also wish to thank Richard Walker for his precious help in translating the original Italian text and editing the English edition.

December 2007

Prologue

Recent years have seen the development of new education and design tools, making it easier for scientists to visualize, modify and personalize their creations. This Prologue provides a guide for readers who wish to build Chua's circuit for themselves and view (or listen to) the enormous variety of patterns the circuit can produce. The prologue is organized in three parts. Each part corresponds to an essential step in understanding the functioning of the Chua circuit. First we show how to build the physical circuit itself. Then we simulate its working, using sounds and music to represent the results of our explorations. Finally we present images of eight navigable environments that the International Journal of Bifurcation and Chaos has used on the covers of some of its issues.

Building the circuit (Building Chua's circuit in twenty two simple steps; coupling the circuit to an oscilloscope)

When trying to understand a physical structure such as Chua's circuit, one of the most effective strategies is "learning by doing". Therefore, in this first part of the prologue, we describe how to build the physical circuit. The "recipe" presented here is based on [Kennedy, 1992] and uses a small number of components, all easily available at low cost. The list includes two operational amplifiers (Op Amps), seven resistors, two capacitors, an inductor and a potentiometer. By connecting the circuit to an oscilloscope users can visualize the values of relevant physical variables. Particularly important is the part of the circuit used to implement Chua's diode, the component that actually produces the chaotic behavior.

Simulation: Simulating Chua's circuit with dimensionless equations

In this section we present a computer program implementing Chua's circuit with dimensionless equations. The software reproduces the mechanisms and processes present in Chua's physical circuit. Attractors are represented using 3D images. The Gallery produced in this way contains 150 patterns. The software allows users to manipulate them in various ways. Possible manipulations include changes in the values of the control parameters, changes in coloring and lighting, moving objects, and changing the size of their basic components. Users can explore different routes to chaos in real time. In this way they gain insights into the way changes in parameter values influence the patterns produced by chaotic behavior. Some of the patterns produced in this way are extremely beautiful.

Chua's Circuit: Exploring chaos through sounds and music

Here we describe a new way of understanding chaos and the forms produced by Chua's circuit - through sounds and music. We present computer programs which use the circuit with dimensionless equations, to generate tonal and atonal music and other sound effects. The programs show the musical potential of chaos. In fact we have even developed new musical instruments, such as the Chua Synth. Instruments like this allow musicians and ordinary people to create tunes and sound effects they could never have imagined. Using them requires no previous knowledge of software or of dynamic systems.

Navigable 3D environments

While building our Gallery we created a series of navigable 3D environments. We have used some of the images from these environments for the covers of the International Journal of Bifurcation and Chaos. It is with these covers that we conclude our prologue.

Building the Circuit

This is the circuit diagram controlled by the following dimensionless equations

$$\begin{cases} \frac{dx}{d\tau} = k\alpha(y - x - f(x)) \\ \frac{dx}{d\tau} = k(x - y + z) \\ \frac{dx}{d\tau} = -k(\beta y + \gamma z) \end{cases}$$

$$f(x) = bx + \frac{1}{2}(a - b)\{|x + 1| - |x - 1|\}$$

Components for Chua's diode

1

2

2 x TL082 operational amplifiers (Op Amp 1, Op Amp 2)

2 x 220Ω resistors (gold, brown, red, red), (R_1 and R_2)

Chua's Diode

2 x 22kΩ resistors (gold, orange, red, red), (R_4 and R_5)

1 x 2.2kΩ resistors (gold, red, red, red), (R_3)

1 x 3.3kΩ resistors (gold, red, orange, orange), (R_6)

Building Chua's Circuit: Components

1 x breadboard

1 x potentiometer (R)

1 x 10 Ω resistor (gold, black, black, brown) (R₀)

1 x 18 mH inductor (L)

1 x 100 nF capacitor (C₂)

1 x 10 nF capacitor (C₁)

Chua's circuit construction step by step

10 nF Capacitor (C_2)

18 mH inductor (L)

1 x 10 Ω Resistor (gold, black, black, brown) R_0

1 Connect inductor L and resistor R_0 into the breadboard. Connect the pin 1 of the inductor to the pin 1 of the resistor. *

2 Connect capacitor C_2 into the breadboard. Connect the pin 2 of R_0 to the pin 1 of C_2 .

* The simplest procedure is to solder a pair of terminals (pins) with a welder and solder. It is even possible to make the connection by twisting the wires together. In this case, it is necessary to ensure that the contact works effectively.

3 Connect the pin 2 of C_2 to the pin 2 of L .

1 3.3 kΩ resistor (gold, red, orange, orange) (R_3 , R_6)

TL082 operational amplifier

7 Connect the first Op Amp 1 D_1 . Connect the pin 1 of D_1 to the pin 1 of C_1 .

8 Connect resistor R_6 . Connect the pin 2 of C_1 to the pin 1 of R_6 .

9 Connect the pin 2 of R_6 to the pin 2 of D_1 .

2 x 22 kΩ resistors (gold, orange, red, red) (R_4 , R_5)

Connect resistor R_5 .
Connect the pin 1 of R_5 to the pin 2 of R_6 .

10

12

Connect the pin 2 of R_4 to the pin 3 of D_1 .

Connect resistor R_4 . Connect the pin 1 of R_4 to pin 1 of C_1 .

11

22 k Ω resistor (gold, orange, red, red) (R₄, R₅)

TL082 operational amplifier

Connect the pin 2 of R₅ to the pin 3 of D₁.

13

14 Connect the Op Amp 2 D₂ .
Connect the pin1 of C₁ to the pin 1 of D₂.

2.2 k Ω resistor (gold, red, red, red) (R_3)

Connect resistor R_3 . Connect the
pin 2 of C_1 to the pin 1 of R_3 .

15

16

Connect the pin 2 of R_3 to the
pin 2 of D_2 .

18 Connect resistor R_1 .
Connect the pin 1 of C_1 to the pin 1 of R_1 .

In order to allow the functioning of the circuit it is necessary to connect two 9V batteries to the Op Amps 1 and 2.

21

Connect the positive pole of the first battery to the pin 2 of the Op Amp 1 and the negative pole to the pin 4 of the Op Amp 1.

22

Connect the positive pole of the second battery to the pin 2 of the Op Amp 2 and the negative pole to the pin 4 of the Op Amp 2.

Changing the values of the potentiometer, a chaotic behaviour is produced.

Connect the circuit to an oscilloscope

After we have built the circuit we can connect the ends of the two capacitors to an oscilloscope. In this way we can display the differences in potential across the capacitors.

Now we can visualize the way the values of the variables change over time.

Alternatively we can display couples of values as a point on a plane. By turning the knob on the potentiometer we can vary the value of resistor R , and watch how the system evolves towards chaos.

Simulating the dimensionless equations for Chua's circuit

Simulation creates a new relationship between the physical and digital worlds - in this case between Chua's physical circuit and a computer simulation of its dynamics. By using a computer we can explore phenomena which it would be difficult or impossible to see in the real world and which only arise in specific circumstances. To this end, we have developed a software tool allowing users to reproduce the patterns produced by Chua's oscillator. The time necessary to become familiar with tool is about 30 minutes. Below we describe how to generate about 150 different patterns produced by the dimensionless equations (DE) for Chua's circuit (see Chapter 3 of the book). Users can view the patterns in a 3D environment where they can examine them from different points of view and explore different routes to chaos.

Proceed as follows.

A. Download the software and start the application

Go to <http://galileo.cincom.unical.it/Chua>

In the "Topics" section, select "Hands-on" and click on "Chaos Explorer". This will start the download.

Once the software has downloaded and decompressed click on run.bat. This will start the application.

WARNING To run, the software needs the latest version of the Java run-time environment. You can download the software from <http://www.java.com/>

B. Load a configuration

On the left of the screen you will see a picture of an attractor. On the right you will see a "glyph". The glyph provides a summary view of the control parameters. For more details see Chapter 3.

Select "Load" from the File menu and load a configuration with a code between DE1 and DE150. Now start the simulation. The patterns you see will be the same as those shown in Table I of Chapter 3.

C. Changing the visual look of the attractor

Use the right mouse button to rotate the attractor

Use the middle button to zoom in

Use editor tools to change the attractor's appearance. For instance you can use "Set Tube Width" in the File menu, to change the width of the tube used to represent the attractor. This will allow you to explore the attractor's fractal nature.

Use the option on the File Menu to change the position of the lights. By default there are lights in front of the attractor, behind the attractor and to the side.

D. Changing the attractor

To change the form of the attractor, change the control parameters. Go to the glyph on the right top corner of the screen, choose a parameter, press the right mouse button and drag it to a new position.

A more precise way of changing the control parameters is to use the File menu to open a parameter window. By changing "Range", you can increase or decrease the sensitivity of the index used to vary the value of the control parameter.

After modifying the parameter values, use "Save" on the File menu to save the new configuration.

You can also save attractors in VRML(Virtual Reality Modeling Language). This lets you use third party modeling software to view the patterns in different ways.

You can also save the image as a .bmp file.

The software's most interesting feature is its ability to iteratively simulate the behavior of dynamic systems. In just a few minutes, you can get a basic idea of chaos and investigate how attractor behavior and patterns vary with changes in the control parameters.

Chua's Circuit: Exploring chaos through sounds and music

A. Introductory notes

Bilotta et al. [2005] proposed the idea of exploring chaos through music and sound. In this part of the Prologue we show how beginners and professional musicians can create music and sounds using the dimensionless equations for Chua's Oscillator. Executable code for some of the programs we use are available for download at <http://galileo.cincom.unical.it/Chua>. We also provides source code, which can use as the basis for their own programs. The site provides examples of musical generated with the software.

B. Components

Basic components needed to generate music with Chua's Oscillator.

Mathematical Models

Generative Codes

Process Codes

External inputs

New musical instruments such as Chua Synth

User Interaction

Representations of Sound and Music

Mathematical models

The mathematical models used here are the dimensionless equations for Chua's Oscillator. These models generate the sequences of numbers used to produce sound and music.

As described in the previous section, users can change the values of the control parameters in real time, using a Glyph. Alternatively they can use a MIDI Input Device. This makes it possible to change both the control parameters and the initial values.

Chapter 3 describes the initial data and control parameters used to generate the base attractors DE1....DE19. Files with these configurations can be downloaded from the web site and modified by the user.

Codes

Generative Codes

The most important codes are :

- a) generative codes used to produce sounds and music
- b) process codes used for digital signal processing. These use the Chua Oscillator to modify the parameters applied during signal processing.
- c) Complex code combining generative and process codes.

Process Codes

New Musical Instruments and Representations

New musical instruments such as the ChuaSynth use chaotic processes to generate music and sound. The design of this kind of instrument is one of the main challenges for Generative Music. The instrument produces all the fundamental frequencies produced by a keyboard instrument. Users can use the qualitative behavior of a Chua attractor (its fundamental pitch) to modify the timbre. By modifying the control parameters, they can explore different routes to chaos. Additional controls allow them to increase or decrease the power of specific frequencies.

Generative Codes and Mathematical Models produce digital signals. To be heard, these signals have to be converted into sounds.

This can be done directly via analog-digital conversion. Alternatively the output can be converted into a MIDI sequence and re-converted into an analog signal. In this second case, users can use additional tools to choose audio channels and instruments.

External input and User Interaction

In some configurations, the software architecture can acquire sound from external sources. These can be either digital or analog.

In some cases they will be generated by sounds (including tunes) and noise. In others they will come from MIDI sources. Signals can also be recorded digitally. They can then be used in the same way as other external sources.

Users can interact with the system by:

- a) Modifying the control parameters;
- b) Using the ChuaSynth to follow a particular route to chaos;
- c) Modifying MIDI parameters to change the music;
- d) Activating or de-activating specific modes of signal processing;
- e) Supplying the system with external inputs.

Usually users interact with the system with a mouse and/or keyboard. Other input may come from pedals and MIDI instruments.

Where needs are complex, these interactions can be simulated by artificial agents.

C. Architectures

In what follows, we describe four architectures underlying the musical software. The first three are designed for Generative Music. The fourth is for signal processing. In each case, we begin by presenting the logical schema for the architecture. We go on to show how the architecture can be implemented in Max patch, a popular software program for music. Finally we describe the user interaction model.

Architecture 1: coding schemes and Generative Music

The system uses various coding schemes to produce tonal and/or atonal music. These include:

- a) Direct production of sounds from the attractor;
- b) Music generation through frequency modulation;
- c) Generation of tonal music via frequency modulation followed by mapping onto musical scales;
- d) Rhythm generation using the maximum and minimum values from time series.

Architecture 1.1 : direct production of sounds from an attractor

We use numerical methods to integrate the equations for the Chua's oscillator. We then use C1 – a generative code – to transform the resulting digital signal into sound. The user can modify the control parameters using external inputs.

Architecture 1.1: an example

Example of a generative architecture using Max™. Users can select an attractor, choose a times series to listen to, modify the control parameters and change the volume of the sound.

Architecture 1.2: generation of atonal music via frequency modulation

The use of external input to change the control parameters

Chua's oscillator

The time series generated by the attractors are used to modulate the frequency of a sine-wave generator.

Frequency Modulation

Sound production

Architecture 1.2: an example

An example of a generate architecture using Max™. Users can modulate a sinusoidal carrier using the times series produced by an attractor. The architecture allows the user to choose one of a set of predefined configurations.

Architecture 1.3: generation of tonal music via frequency modulation and mapping onto musical scales

The use of external input to change the control parameters

Chua's oscillator

Code to transform time series into numbers for use as a MIDI sequence.

Music produced from MIDI sequences

The time series generated by the attractors are mapped to numerical values. These numbers are used as MIDI codes.

Architecture 1.3: an example

Example of a generative architecture based on MaxTM. The time series generated by the attractors are mapped to numerical values. These numbers are used as MIDI codes. The architecture makes it possible to listen to two streams of numbers simultaneously.

Architecture 1.4: generation of rhythms using the minimum and maximum values of the time series

We use a sampled drum sound to represent the minimum value and a second drum sound for the maximum. Both are represented as MIDI codes. With this scheme, it is interesting to observe the emergence of new rhythms.

Architecture 1.4: an example

Example of a generative architecture based on Max[™]. The user can manipulate the code to identify the maxima and minima in time series which will then be used as MIDI codes.

User Interaction

In all the first family of architectures we have presented, users interact with the system by:

- changing the parameters of the mathematical model;
- activating or deactivating codes with a button;
- modifying the MIDI codes.

The figure depicts the Max Patch user interface used to modify the control parameters and to edit the MIDI code.

Architecture 2: generative Music with the ChuaSynth

The use of external input to change the control parameters

Chua's oscillator

Code to transform the time series into a sequences of notes for ChuaSynth

Music from ChuaSynth

ChuaSynth

Generation of tonal music through frequency modulation and mapping to musical scales.
The time series generated by the attractors are mapped onto a musical scale. The relative frequencies are then played, using ChuaSynth.

External input: the user can modify the ChuaSynth parameters

Architecture 2: an example

An example of Architecture 2, based on Max[™]. This architecture uses ChuaSynth. Users can modify both the control parameters of the differential equations and the x settings of the ChuaSynth. In this way they can create memories using the voices provided by ChuaSynth.

Architecture 3: mapping systems using input melodies

Architecture 3: an example

This is the Max Patch user interface used to implement the Architecture 3. The functional blocks are easy to recognize. To describe the generation of music from Chua attractors we use the metaphor of a "music-making triangle" [Bilotta et al., 2005].

In this triangle one of the vertices represents a mathematical structure, for example the equations for Chua's Oscillator, as visualized in the first figure.

This second figure depicts an example of a Max Patch for representing a Mathematical Model.

The second vertex represents the transformation of the times series into a MIDI sequence. The third vertex represents the transformation of the two inputs (from the mathematical system and the musician) into a final sequence. The two figures show Max Patches used to create the codes required by these vertices of the music triangle.

Generative Architecture

In this Architecture 3, we can introduce a system that takes sound from an external source (e.g. a MIDI keyboard, a musical instrument such as an electric guitar). This makes it possible to control the fundamental pitch of the composition. The mapping system can use different MIDI events. It is also possible to change keys and instruments (or combinations of instruments). External controls allow users to modify the control parameters in real time. In this way, musicians can interact with the instrument and exploit the system's chaotic behavior as the basis for musical improvisation. The figure shows a Max Patch used to implement this functionality.

Architecture 4: Signal Processing

When musicians interact with the generative algorithm, they can use the external signal to control the generative process in real time.

In this architecture, a synthesizer module combines an external electrical signal with the signal generated by Chua's circuit; a rhythm generator maps the output from the circuit onto sampled drum beats; two signal processing modules combine the external signal with the signal from Chua's circuit. For this, they use convolutions/ring modulations together with a delay module that uses the period of the Chua's Oscillator to modulate lag in the output.

P1 is the pitch detection process.

P2 is the combined generation process with the ChuaSynth.

P3 represents the convolution process.

P4 is the flanger process.

P5 is the rhythm process.

Architecture 4: an example

The figure shows the Max Patch used to implement the architecture 4. To use the application, you have to click on the on/off radio sign. This switches on the sound board. Then choose one of the presets.

In the example, we apply a pitch detection operator to the input signal (from a microphone or an electric instrument).

The images, designed as Max Patches, show the main steps in the pitch detection process. If you do not have a microphone or an electric instrument you can use the sine-wave generator to create an input signal. You can choose any frequency between 50 e 1800 Hz. The pitch detector analyzes the signal and converts it into a MIDI pitch. This information is then re-converted into sound.

This signal is taken as an input by the three oscillators in the Chua Synth. The oscillators synthesize a chaotic sound. To do this they modify the alpha, beta and gamma parameters of Attractor 1, as produced by the Chua circuit. The Timbre Distortion parameter acts on the a and b parameters. This makes it possible to shift from perfectly periodic to completely chaotic spectra.

This is when the true generative process comes into play. The process works by modulating the frequency detected by "pitch".

The system provides four modulation oscillators, all based on the same attractor. To choose the attractor, use the panel on the right. Additional controls make it possible to vary the values of the control parameters for the attractor. To view its behavior, click on "p graphics".

Although attractors display chaotic behavior they always have a "characteristic frequency". This is calculated by recording a minute of signal, taking 65536 samples per time window and applying the FFT.

Once the characteristic frequency is known, it is possible to scale the speed of the signal, by acting on the k parameter. A similar technique regulates the attractor's characteristic beat frequency. To do this, go to the right panel, choose a value for BPM, and double click on "tapping for beat". The period between two beats will be set to the characteristic period for the attractor. This technique allows musicians to choose their own tempo and ensure that the attractor will always be in synchronization.

The four modulation circuits all use the same attractor. However each circuit generates its own rhythm.

Rhythm patterns are generated in the pale blue rectangle in the lower right of the screen. Sounds are produced every time the attractor's trajectory reaches a maximum or a minimum. The sounds used to represent rhythm were recorded from an acoustic drum. The pitch is derived from the value of the attractor when it reaches a maximum or minimum. To observe the coding process more closely click on "p BeatBox".

We now examine signal processing (shown in the orange and yellow panels). The panel allows users to choose between two different kinds of convolution and ring modulation. To observe the steps in greater detail click on "p processing"

The stereo lag control allows users to define lag, using the time between two relative maxima or two relative minima.

Navigable 3D environments

(Published in February 2005)

The cover is inspired by the idea that human cognition, and the surrounding world, are chaotic dynamical systems similar to Chua's oscillator. The architectural components that manage both the cognitive functions and the world around us are articulated into emergent and self-organized patterns, at different scales, even if the matter of determined physical or psychological or social processes could be different. Chua's attractors could well identify these processes. A man from the future that realizes dynamical cognitive processes, observes the attractor he holds in his hand and listens to the music that comes from it. All around, the cities are built on worlds that are represented by other Chua's attractors – perhaps, Chua's approach could be very helpful in understanding some key-topics of the science of complexity.

(Published in March 2006)

The Language of Chaos hypothesizes that it is possible to manage chaos and to detect, in chaotic behaviour, different levels of organization as well as distinctive components such as phonemes, morphemes and a generative syntax, as in natural human languages.

The central figure on the cover represents a network of morphemes, while the other objects are models of the time-series sequencing method the authors have used to analyze the behaviour of Chua chaotic attractors. The language of chaos is a new way of understanding the structure and function of complexity. It could become the foundation for explaining what appear to be common denominators in many different real-world biological, technological and social phenomena, opening up a scenario in which the essential physical nature of things possesses the same organization. The basic idea is to consider the mental-physical binomial as a gradual continuum in which expressive languages differ only superficially, but remain substantially analogous in their organization structures and essential components, comprising living and cognitive matter.

(Published in January 2007)

The cover image suggests a metaphor of the “Gallery of Chua Attractors”. It is a museum room in which the art objects are represented by attractors, which stand for non-traditional sculptures, disposed in the environment. Many other pictures are hung on the wall, pictures that portray characteristics which have been caught inside the fractal nature of the strange attractors.

The environment can be studied on the net by exploring the objects, getting information on the attractors, obtaining data for their reproduction, and changing their parameters in order to modify their patterns. New forms of interaction and of Computer Art is growing, allowing researchers to explore, manipulate, learn in a very interesting way and improve theoretical and experimental investigations of chaotic systems.

(Published in February 2007)

How can we represent the parameter space for Chua dynamical systems?

We have hypothesised that many features of this space can be visualized by using the terrestrial metaphor, presenting features of the landscapes we can observe in nature.

The cover image shows an imaginary parameter space of Chua attractors. In this strange land, attractors stand for non-traditional buildings, disposed in the environment. They are everywhere, sometimes alone as if they were islands in the sea, sometime concentrated in a building, as if they belonged to the building. A sky line made of pyramids - which are generated by Chua systems as well - is in the background. This imaginary space has been realized as a 3d environment, so that it can be explored as if it were real. Many issues related to design (such as the amount of polygons of each attractor, the number of elements in the space, the reduction of the complexity of each attractor) have been solved in order to create this environment.

In this city of attractors many activities related to the study and to the interaction with Chua systems are going to be implemented in order to permit users not only to learn complex issues related to Chaos but also to have fun like in an experimental museum.

(Published in March 2007)

In our application of different methods of searching for chaotic attractors, we analyzed a great variety of routes leading to chaos, and identified qualitative changes in chaotic behavior associated with changes in parameter values. This enabled us to extrapolate meaning from huge volumes of data, yet it also required the exercise of human judgment to identify patterns that were particularly beautiful or novel in terms of shape, size, and evolution over time.

The cover picture was obtained by representing both the parameter space and the space of the phases of dimensionless Chua attractors, through the use of innovative visualisation techniques.

The results were interpreted artistically, by means of the realisation of a multi-dimensional space in which the parametrical structure gives life to chaotic forms which “grow” on imaginary trees.

(Published in April 2007)

By exploring parameter space, we can obtain an infinite variety of Chua attractors, which we can visualize in a number of different representations. In the cover picture we show example of morphogenetic dynamics, in phase space, generated by an exploration of the parameter space for the Chua cubic system. In the central panel, we see a set of development trajectories displaying evident clustering. In this image, we see some of the qualitative changes in patterns produced by variations in parameter values. The other panels show a series dynamic views of these changes, demonstrating the graphical potential of the model. In graphical terms, what we see is a three-dimensional growth process. In other words, our images allow us to visualize changes in the 3D structure of the patterns and to represent them in clustered structures. The growth dynamics of these structures follow an evolutionary path which is largely smooth and fluid but which also contains abrupt transitions. Although the patterns generated depend on a number of general rules, the actual evolutionary path followed by a specific system depends on the initial conditions and on the choice of parameter values and is completely unpredictable.

(Published in May 2007)

Hausdorff distance provides a useful metric for exploring the parameter space for Chua's attractor. The cover image shows a set of plots representing Hausdorff distances between a Chua single scroll attractor (the SS1 attractor), used as a reference, and a set of attractors, generated from the base attractor. Each plot is obtained by starting from SS1 and varying two parameters at a time within a range of values obtained from previous studies of bifurcation maps for Chua single scroll equations. For purposes of visualization, we mapped the distances obtained to an ad hoc color code. This technique makes it easy to see the fine grain structure of the two-dimensional space and to infer and test candidate laws of morphogenesis. The findings – which suggest the existence of many different modes of propagation – represent a new approach in artificial evolutionary modeling.

(Published in June 2007)

The cover for this issue of the journal shows a region of the parameter space for Chua attractors generated by a n-scroll system. Within this space, users equipped with special glasses and data gloves, interact dynamically with unexplored regions of chaos. Using the 3D environment they can move, work and make observations as freely as if they were in a real physical environment. Within the environment, they have enormous freedom to choose the way they will interact with the attractors: they can select the dimensions of parameter space used to create the spaces they are interacting with, observe these spaces from different view points, take snapshots of the environment, and use the data gloves to interact with and change the shapes they see.

In the environment, users acquire their awareness of chaos directly through their auditive, visual, and tactile perceptions. The new tool expands our ability to understand chaos and its internal organization. Once, these were purely imaginary spaces. Today, through the ad hoc tools we have developed during our work, we can see them with our own eyes.

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