MMULTIDIMENSIONAL SECOND ORDER STOCHASTIC PROCESSES

SERIES ON MULTIVARIATE ANALYSIS

Editor: M M Rao

Published

Vol. 1: Martingales and Stochastic Analysis

J. Yeh

**Multivariate
Analysis**
•— Vol. 2 —•

MULTIDIMENSIONAL SECOND ORDER STOCHASTIC PROCESSES

Yûichirô Kakihara

*Department of Mathematics
University of California, Riverside*

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Kakihara, Yûichirô.

Multidimensional second order stochastic processes / Yûichirô Kakihara.

p. cm. -- (Series on multivariate analysis ; vol. 2)

Includes bibliographical references and indexes.

ISBN 9810230001

1. Stochastic processes. I. Title. II. Series.

QA274.K328 1996

519.2--dc21

96-37165

CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 1997 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

This book is printed on acid-free paper.

Printed in Singapore by Uto-Print

To my family

Keiko, Noriko and Satoko

This page is intentionally left blank

Preface

Univariate stationary stochastic processes in the weak sense were initiated by Khintchine in 1934, and since then the theory has been developed by many authors. In many applications, however, nonstationary processes play an important role. As an extension of stationarity Loève, in middle 1940s, introduced harmonizability and, in 1959, Rozanov gave another harmonizability, where Rao distinguished these two concepts by calling them strong and weak harmonizabilities in 1982. Another extension is to consider multidimensional stochastic processes. In late 1950s Wiener and Masani developed such a theory for stationary stochastic processes.

This book is concerned with multidimensional (mainly *infinite dimensional*) nonstationary (especially *harmonizable*) second order stochastic processes on (mainly) a locally compact abelian group. Since harmonizable processes are the Fourier transforms of Hilbert space (typically $L^2(\mu)$) valued measures, our treatment uses functional analysis as well as harmonic analysis instead of basing it on probability theory alone.

Here is a brief description of each chapter. In Chapter I, a historical survey of stationary and nonstationary processes as well as multidimensional and other extensions are given. In Chapter II, normal Hilbert $B(H)$ -modules are considered in detail, where H is a (separable) complex Hilbert space and $B(H)$ is the algebra of all bounded linear operators on H . This kind of spaces is useful to describe infinite dimensional second order stochastic processes and is essentially the same as the spaces of Hilbert-Schmidt class operators between Hilbert spaces.

In Chapter III, we investigate stochastic measures and operator valued bimeasures. Let $L_0^2(\Omega; H)$ be the Hilbert space of all H -valued random variables on a probability space $(\Omega, \mathfrak{F}, \mu)$ which are μ -square integrable. This space is a typical example of a normal Hilbert $B(H)$ -module and an $L_0^2(\Omega; H)$ -valued measure can be regarded as an infinite dimensional stochastic measure. Such a measure induces a bimeasure with values in a subspace of $B(H)$ and, in a certain sense, the converse is also true. In Chapter IV, we give the fundamental theory of multidimensional stochastic processes. After defining stationarity, harmonizability and other nonstationarities, we discuss their integral representations and interrelations. As basic topics we consider series representations, moving average representations, approximation and convergence, and subordination.

In Chapter V, some special topics are studied such as Wold and Cramér decompositions, and KF-processes. Uniformly bounded linearly stationary processes and periodically correlated processes are of interest and are also treated in some detail.

Some basic results on isotropic processes, processes on hypergroups, and processes on locally compact groups are stated without proof since a greater preparation is needed to give a full treatment of these topics. Finally in Chapter VI, applications are made from probabilistic and statistical points of view to prediction problems, Kalman filter, sampling theorems and the strong law of large numbers.

Each chapter has the "Bibliographical notes" section assigning proper, and hopefully accurate, credits to authors whose contributions supported this book.

A numbering system of this book should be mentioned. An item such as III.4.5 denotes the fifth in Section 4 of Chapter III. In a given chapter, only the section and item numbers are used, and in a given section only the item number is used.

This book was suggested by Professor M. M. Rao of University of California at Riverside (UCR). He kindly read the manuscript and gave criticism, advice and encouragement. The author would like to express his sincere gratitude to Professor Rao. He also thanks the hospitality of the Department of Mathematics of UCR where he could finish writing and typing this book during the stay.

Yûichirô Kakihara
Riverside, California
September, 1996

Table of Contents

Preface	iii
Chapter I. Introduction and preliminaries	1
1.1. Stationary processes	1
1.2. Harmonizable processes	4
1.3. Multidimensional and other extensions	9
Bibliographical notes	15
Chapter II. Hilbert modules and covariance kernels	17
2.1. Normal Hilbert $B(H)$ -modules	17
2.2. Submodules, operators and functionals	21
2.3. Characterization and structure	26
2.4. Positive definite kernels and reproducing kernel spaces	32
2.5. Harmonic analysis for normal Hilbert $B(H)$ -modules	46
Bibliographical notes	52
Chapter III. Stochastic measures and operator valued bimeasures	53
3.1. Semivariations and variations	53
3.2. Orthogonally scattered dilations	74
3.3. Gramian orthogonally scattered dilations	86
3.4. The spaces $L^1(F)$ and $L^2(F)$	107
3.5. The spaces $\mathfrak{L}^1(\xi)$ and $\mathfrak{L}^2(M)$	119
3.6. Riesz type theorems	129
3.7. Convergence	136
Bibliographical notes	145
Chapter IV. Multidimensional stochastic processes	148
4.1. General concepts	148
4.2. Stationary processes	151
4.3. Harmonizable processes	155
4.4. V -bounded processes	166
4.5. Cramér and Karhunen classes	173
4.6. Series representations	178
4.7. Moving average representations	183

4.8. Approximation and convergence	184
4.9. Subordination	192
Bibliographical notes	200

Chapter V. Special topics **203**

5.1. Wold decompositions	203
5.2. Cramér decompositions	208
5.3. The KF-class	212
5.4. Uniformly bounded linearly stationary processes	218
5.5. Periodically correlated processes	223
5.6. Final remarks	234
(1) Isotropic processes	234
(2) Processes on hypergroups	237
(3) Processes on locally compact groups	241
Bibliographical notes	245

Chapter VI. Applications **248**

6.1. Prediction problems	248
6.2. Kalman filter	259
6.3. Sampling theorems	275
6.4. Strong laws of large numbers	285
Bibliographical notes	293

References **296**

Indices **316**

Notation index	316
Author Index	322
Subject index	326