Fork Algebras in Algebra, Logic and Computer Science

Advances in Logic

Series Editor: Dov M Gabbay FRSC FAvH
Department of Computer Science
King's College London
Strand, London WC2R 2LS
UK
dg@dcs.kcl.ac.uk

Published

- Vol. 1 Essays on Non-Classical Logic
 by H. Wansing
- Vol. 2 Fork Algebras in Algebra, Logic and Computer Science
 by M. Frias

Advances in Logic – Vol. 2

Fork Algebras in Algebra, Logic and Computer Science

Marcelo Fabián Frias

University of Buenos Aires, Argentina

World Scientific

New Jersey • London • Singapore • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Frias, Marcelo Fabián, 1968–

Fork algebras in algebra ; logic and computer science / Marcelo Fabián Frias.

p. cm. -- (Advances in logic ; v. 2)

Includes bibliographical references and index.

ISBN 9810248768 (alk. paper)

I. Computer science -- Mathematics. 2. Logic, symbolic and mathematical. I. Title.
II. Series.

QA76.9.M35 F75 2002

004'.01'51--dc21

2002066193

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 2002 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

Printed in Singapore by Uto-Print

to Norma, Taís and Gastón

This page is intentionally left blank

Preface

This book is the result of the research carried on by the author and some of his colleagues from 1995. Fork algebras, the subject of this book, had their origin in the early 90s as part of a formalism capable of dealing with the process of program specification and development. The contents of the book fall in with what are called *Relational Methods in Computer Science*.

As usually happens, applied research led to problems of a theoretical nature which were undertaken by the author and are the core of this book. Problems such as *finite axiomatizability* or *axioms independence* (Sections 4.1 and 4.2) naturally arose when investigating the relational semantics of the fork calculus.

Relational proof systems for various logics (classical, modal and multi-modal) (Sections 5.2, 6.5, 6.7) besides providing relational deductive calculi for these logics, allow us to assess the expressive power of the fork calculus and establish the foundations for a relational formalism for system specification.

Finally, in Section 7.5 we present the foundations for a relational calculus for program specification and derivation that allow us to specify and calculate program design strategies.

The author wishes to thank Armando Haeberer and Roger Maddux, who were his Ph.D. advisors and co-authors of several results in this book. Gabriel Baum has to be thanked not only as a colleague and co-author of several papers, but also as a constant source of friendship and advice. Tom Maibaum suggested that these results could be put in the form of a book. The members of the RelMiCS (*Relational Methods in Computer Science*) group are thanked for their always useful comments and criticisms. Finally,

the author wishes to thank the Department of Informatics, PUC-RIO; LI-FIA, National University of La Plata; and the Department of Computer Science, School of Sciences, University of Buenos Aires, for providing an always stimulating environment.

Contents

Preface	vii
Chapter 1 Introduction and Motivations	1
1.1 Software Specification, Binary Relations and Fork	1
Chapter 2 Algebras of Binary Relations and Relation Algebras	5
2.1 History and Definitions	5
2.2 Arithmetical Properties	12
Chapter 3 Proper and Abstract Fork Algebras	19
3.1 On the Origin of Fork Algebras	19
3.2 Definition of the Classes	21
3.3 Arithmetical Properties	26
Chapter 4 Representability and Independence	37
4.1 Representability of Abstract Fork Algebras	38
4.2 Independence of the Axiomatization of Fork	43
Chapter 5 Interpretability of Classical First-Order Logic	49
5.1 Basic Definitions	49
5.2 Interpreting <i>FOLE</i>	51
Chapter 6 Algebraization of Non-Classical Logics	73
6.1 Basic Definitions and Properties	75
6.2 The Fork Logic <i>FL</i>	76

6.3	Modal Logics	78
6.4	Representation of Constraints in FL	80
6.5	Interpretability of Modal Logics in FL	81
6.6	A Proof Theoretical Approach	86
6.7	Interpretability of Propositional Dynamic Logic in FL	91
6.8	The Fork Logic FL'	102
6.8.1	Syntax of FL'	102
6.8.2	Semantics of FL'	102
6.9	A Rasiowa–Sikorski Calculus for FL'	104
6.9.1	The Deduction System for FL'	105
6.9.2	Soundness and Completeness of the Calculus FLC	107
6.9.3	Examples of Proofs in the Calculus FLC	112
6.10	A Relational Proof System for Intuitionistic Logic	115
6.10.1	Intuitionistic Logic	115
6.10.2	Interpretability of Intuitionistic Logic in FL'	117
6.10.3	A Fork Logic Calculus for Intuitionistic Logic	121
6.10.3.1	Example	124
6.11	A Relational Proof System for Minimal Intuitionistic Logic	126
6.12	Relational Reasoning in Intermediate Logics	132
6.12.1	Method 1	132
6.12.2	Method 2	133
6.12.3	Method 3	137
Chapter 7 A Calculus for Program Construction		139
7.1	Introduction	139
7.2	Filters and Sets	141
7.3	The Relational Implication	143
7.4	Representability and Expressiveness in Program Construction	149
7.5	A Methodology for Program Construction	150
7.6	Examples	158
7.6.1	First Example	159
7.6.1.1	Finding the Minimum Element in a List	167
7.6.1.2	Finding the Minimum Common Ancestor	168
7.6.2	Second Example	174
7.6.2.1	Finding the Contiguous Sublists of Maximum Sum	184
7.6.2.2	Finding the Longest Plateau	187
7.7	A $D\&C$ Algorithm for $MAXSTA$	192

7.8 Comparison with Previous Work	203
Bibliography	207
Index	215