Defects of Properties in **Mathematics**

Quantitative Characterizations

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Quantitative Characterizations

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To our lovely wives Olimpia Ban and Rodica Gal

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Preface

The main aim is to reveal a method of research in mathematics, called by us quantitative study of the defect of property, which can be used in various fields of mathematics. Our viewpoint over the mathematical entities is that of an analyst, even if they belong to algebra, geometry, topology or logic. We examine in this monograph in a systematic way, the quantitative characterizations of the "deviation from a (given) property", by our terminology called "defect of property", in: Set Theory, Topology, Measure Theory, Real Function Theory, Complex Analysis, Functional Analysis, Algebra, Geometry, Number Theory and Fuzzy Mathematics. We also present a great variety of applications and open problems. To our knowledge, it is the first time in literature that a book has systematically studied this direction of research. This book is interdisciplinary in mathematics and contains the research of both authors over the past six years in these subjects. It also references most of the works of other main researchers in these areas. Each chapter can be read independently. The introduced concepts are simple and the proving methods are rather elementary, that is, making this material accessible to undergraduate and graduate students and researchers. The large spectrum covered by the topic, makes impossible to have a complete bibliography, which is only introductory and depends on the authors' preferences. The book is of wide audience and it is good for researchers, undergraduate and graduate students, courses and seminars.

Oradea, August 1, 2001

The authors

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Contents

Preface	vii
Chapter 1 Introduction	1
1.1 General Description of the Topic	1
1.2 On Chapter 2: Defect of Property in Set Theory	3
1.3 On Chapter 3: Defect of Property in Topology	8
1.4 On Chapter 4: Defect of Property in Measure Theory	18
1.5 On Chapter 5: Defect of Property in Real Function Theory	24
1.6 On Chapter 6: Defect of Property in Functional Analysis	31
1.7 On Chapter 7: Defect of Property in Algebra	45
1.8 On Chapter 8: Miscellaneous	50
Chapter 2 Defect of Property in Set Theory	57
2.1 Measures of Fuzziness	57
2.2 Intuitionistic Entropies	67
2.3 Applications	76
2.3.1 Application to determination of degree of interference	76
2.3.2 Application to description of the performance of systems	81
2.3.3 Application to digital image processing	84
2.4 Bibliographical Remarks	86
Chapter 3 Defect of Property in Topology	87
3.1 Measures of Noncompactness for Classical Sets	87
3.2 Random Measures of Noncompactness	95
3.3 Measures of Noncompactness for Fuzzy Subsets in Metric Space	98

3.4	Measures of Noncompactness for Fuzzy Subsets in Topological Space	111
3.5	Defects of Opening and of Closure for Subsets in Metric Space	125
3.6	Bibliographical Remarks and Open Problems	127
Chapter 4	Defect of Property in Measure Theory	131
4.1	Defect of Additivity: Basic Definitions and Properties	131
4.1.1	Application to calculation of fuzzy integral	142
4.1.2	Application to best approximation of a fuzzy measure	145
4.1.3	A metric on the family of fuzzy measures	146
4.2	Defect of Complementarity	148
4.3	Defect of Monotonicity	164
4.4	Defect of Subadditivity and of Superadditivity	173
4.5	Defect of Measurability	181
4.6	Bibliographical Remarks	184
Chapter 5	Defect of Property in Real Function Theory	185
5.1	Defect of Continuity, of Differentiability and of Integrability	185
5.2	Defect of Monotonicity, of Convexity and of Linearity	193
5.3	Defect of Equality for Inequalities	200
5.4	Bibliographical Remarks and Open Problems	208
Chapter 6	Defect of Property in Functional Analysis	211
6.1	Defect of Orthogonality in Real Normed Spaces	211
6.2	Defect of Property for Sets in Normed Spaces	234
6.3	Defect of Property for Functionals	246
6.4	Defect of Property for Linear Operators on Normed Spaces	252
6.5	Defect of Fixed Point	260
6.6	Bibliographical Remarks and Open Problems	273
Chapter 7	Defect of Property in Algebra	277
7.1	Defects of Property for Binary Operations	277
7.2	Calculations of the Defect of Property	289
7.3	Defect of Idempotency and Distributivity of Triangular Norms	293
7.4	Applications	300
7.5	Bibliographical Remarks	304
Chapter 8	Miscellaneous	305
8.1	Defect of Property in Complex Analysis	305

8.2 Defect of Property in Geometry 307

8.3 Defect of Property in Number Theory 320

8.4 Defect of Property in Fuzzy Logic 327

8.5 Bibliographical Remarks and Open Problems 334

Bibliography 337

Index 349