GEOMETRIC CONTROL AND NONSMOOTH ANALYSIS

This page intentionally left blank

Series on Advances in Mathematics for Applied Sciences – Vol. 76

GEOMETRIC CONTROL AND NONSMOOTH ANALYSIS

*In Honor of the 73rd Birthday of H. Hermes
and of the 71st Birthday of R.T. Rockafellar*

Edited by

Fabio Ancona

Università di Bologna, Italy

Alberto Bressan

Penn State University, USA

Piermarco Cannarsa

Università di Roma "Tor Vergata", Italy

Francis Clarke

Université Claude Bernard Lyon I, France

Peter R Wolenski

Louisiana State University, USA

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Geometric control and nonsmooth analysis / edited by Fabio Ancona ... [et al.].

p. cm. -- (Series on advances in mathematics for applied sciences ; v. 76)

Includes bibliographical references and index.

ISBN-13: 978-981-277-606-8 (hardcover : alk. paper)

ISBN-10: 981-277-606-0 (hardcover : alk. paper)

1. Control theory--Research. 2. Nonsmooth optimization--Research.

3. Systems engineering--Research. I. Ancona, Fabio, 1964--

QA402.3.G436 2008

515'.642--dc22

2008017673

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 2008 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

Printed in Singapore by B & JO Enterprise

PREFACE

Applied mathematics and engineering abound with situations in which one wishes to influence the behavior of systems that are modeled as trajectories of differential equations. This has given rise to the large and multivalent subject known as *control*, the mathematical theory of which is thought of as having been born in the 1950's. The goal of controlling a system can be (roughly speaking) of two sorts: for optimality (as in the minimization of a cost), or for positional purposes (for example, driving the state to an equilibrium). The first of these is directly in the line of thought of the classical calculus of variations, while the second has its origins primarily in engineering systems design.

An important feature of control theory in recent decades has been the convergence of these two types of control at the level of the underlying theory. Thus the distinction between them has blurred in recent decades. A principal reason for this has been the development of tools which successfully address issues of both types, especially those stemming from geometry and from nonsmooth analysis.

The interaction between geometry and control theory rests upon the use of classical differential-geometric tools, in particular Lie algebras of vector fields, free and nilpotent Lie algebras, filtrations, symplectic geometry, exterior differential systems, infinite dimensional manifolds. Also involved are such concepts of analytic geometry and dynamical systems as subanalytic sets, stratifications, center and stable manifolds, among others.

Somewhat more recently, the use of nonsmooth analysis has revealed itself to be essential in both optimal and positional control. Nonsmooth calculus and its related circle of ideas (variational principles, penalization, differential inclusions, generalized tangency and normality...) can now be viewed as a relatively mature subject. The theory has been applied to control issues with great effect, in combination with such tools as viscosity solutions, dynamic programming, Hamilton-Jacobi inequalities, duality, discontinuous feedbacks, convexity and semiconcavity, and nonsmooth Lyapunov functions.

Geometric methods and nonsmooth analysis have together given rise to many recent developments in control theory. Among these are: new and unified formulations of necessary conditions in optimal control, the synthesis and classification of optimal or stabilizing controls, and new techniques for optimal and stabilizing feedback design, in particular sliding modes.

The conference *Geometric Control and Nonsmooth Analysis*, which took place in Rome in June 2006 at the Istituto Nazionale di Alta Matematica (INdAM), celebrated the leading contributions to control theory of two eminent researchers, Henry Hermes and R. T. Rockafellar (often known as Hank and Terry to their many friends). The topics covered a wide range of the issues mentioned above, from the calculus of variations to optimal control, from controllability to stability, from invariance to impulse systems, from approximation to feedback design. This volume, which assembles the contributions of the speakers of the conference, provides an overview which illustrates well the vitality, the range, and the importance of the subject that is control theory.

Soon after the conference, we learnt that Wijesuriya Dayawansa, whom we had just seen in Rome, had abruptly passed away. This volume is dedicated to his memory.

Fabio Ancona

Alberto Bressan

Piermarco Cannarsa

Francis H. Clarke

Peter W. Wolenski

Bologna, Italy

Penn State, USA

Roma, Italy

Lyon, France

Baton Rouge, USA

April 2008

PROFESSOR HENRY HERMES
UNIVERSITY OF COLORADO AT BOULDER

PROFESSOR R. TYRRELL ROCKAFELLAR
UNIVERSITY OF WASHINGTON

SCIENTIFIC COMMITTEE

- A. Bressan – Penn State University, USA
 P. Cannarsa – Università di Roma “Tor Vergata”, Italy
 F. H. Clarke – Université Claude Bernard Lyon I, France
 G. Stefani – Università di Firenze, Italy

ORGANIZING COMMITTEE

- F. Ancona – Università di Bologna, Italy
 M. Kowski – Arizona State University, USA
 B. Piccoli – IAC - CNR, Roma, Italy
 P. R. Wolenski – Louisiana State University, USA

SUPPORTED BY

- INdAM - Istituto Nazionale di Alta Matematica “F. Severi”.
- MIUR PRIN 2004 “Controllo, Ottimizzazione e Stabilità di Sistemi non Lineari: Metodi Geometrici ed Analitici”
- MIUR PRIN 2005 “Metodi di viscosità, metrici e di teoria del controllo in equazioni alle derivate parziali nonlineari”
- National Science Foundation grant “Conference on Optimal Control and Nonsmooth Analysis”, DMS 0612807.
- Dipartimento di Matematica, Università di Roma “Tor Vergata”.

CONTENTS

Preface	v
Conference Committees	viii
Multiscale Singular Perturbations and Homogenization of Optimal Control Problems	1
<i>O. Alvarez, M. Bardi and C. Marchi</i>	
Patchy Feedbacks for Stabilization and Optimal Control: General Theory and Robustness Properties	28
<i>F. Ancona and A. Bressan</i>	
Sensitivity of Control Systems with Respect to Measure- Valued Coefficients	65
<i>Z. Artstein</i>	
Systems with Continuous Time and Discrete Time Components	82
<i>A. Bacciotti</i>	
A Review on Stability of Switched Systems for Arbitrary Switchings	100
<i>U. Boscin</i>	
Regularity Properties of Attainable Sets under State Constraints	120
<i>P. Cannarsa, M. Castelpietra and P. Cardaliaguet</i>	
A Generalized Hopf–Lax Formula: Analytical and Approx- imations Aspects	136
<i>I. Capuzzo Dolcetta</i>	
Regularity of Solutions to One-Dimensional and Multi- Dimensional Problems in the Calculus of Variations	151
<i>F.H. Clarke</i>	

Stability Analysis of Sliding Mode Controllers <i>F.H. Clarke and R.B. Vinter</i>	164
Generalized Differentiation of Parameterized Families of Trajectories <i>M. Garavello, E. Girejko and B. Piccoli</i>	177
Sampled-Data Redesign for Nonlinear Multi-Input Systems <i>L. Grüne and K. Worthmann</i>	206
On the Definition of Trajectories Corresponding to Generalized Controls on the Heisenberg Group <i>P. Mason</i>	228
Characterization of the State Constrained Bilateral Minimal Time Function <i>C. Nour</i>	243
Existence and a Decoupling Technique for the Neutral Problem of Bolza with Multiple Varying Delays <i>N.L. Ortiz</i>	248
Stabilization Problem for Nonholonomic Control Systems <i>L. Rifford</i>	260
Proximal Characterization of the Reachable Set for a Discontinuous Differential Inclusion <i>V.R. Rios and P.R. Wolenski</i>	270
Linear-Convex Control and Duality <i>R.T. Rockafellar and G. Goebel</i>	280
Strong Optimality of Singular Trajectories <i>G. Stefani</i>	300
High-Order Point Variations and Generalized Differentials <i>H. Sussmann</i>	327
List of Participants	359
Author Index	361