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To Lara Celine and Anne Sophie (H.M.-O.)

To My Parents (T.R.)

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Preface

Although phase transitions of particle physics appear as exotic phenomena from the viewpoint of daily-life physics, their occurrence in the early universe has far reaching consequences for the status of the universe nowadays and makes these transitions conceptually interesting. "Little bangs" in heavy-ion collisions may reproduce these exotic states of matter in lab-experiments, since the available energies in these collisions are meanwhile sufficiently high to produce these states for a very short instant of time. The aim of the book is to derive the phase structure of particle physics by applying standard thermodynamic concepts to particle physics.

The book is based on a review about *Phase Transitions in Quantum Chromodynamics* by one of the authors (H. M.-O.), published in Rev. Mod. Phys. 68 (1996) 473, and on a series of lectures about *Phase Transitions and Critical Phenomena with Applications to Particle Physics* at the Universities of Wuppertal and Heidelberg. It was extended to include results on the phase structure of the electroweak part of the standard model, as well as further theoretical developments such as perturbation theory on the lattice at zero and finite temperature, dimensional reduction, flow equations, gap equations and various resummation techniques.

For a detailed understanding of the phase structure of quantum chromodynamics and the electroweak standard model, approximations in analytical and numerical calculations are unavoidable. The aim of the book is to explain these methods and to clearly specify the conditions under which the approximations are under control. The analytical part covers methods both in a continuum regularization and on a spacetime lattice. Since the lattice plays a distinguished role as a nonperturbative treatment of the phase structure of the standard model, it makes up the larger part

of the book. In the chapter on numerical methods we focus on criteria for an appropriate interpretation of numerical data rather than on details of algorithms to produce them. Where possible we try to bridge the gap between different approaches to one and the same problem, for example between a phenomenological description of first-order phase transitions and rigorous results about their finite-size scaling analysis, or between lattice and continuum approaches, or between effective models and first-principle calculations. The presentation also contains more recent topics which cannot be found in other textbooks so far. To these belong generalized series expansions, dimensional reduction, resummation techniques and certain realizations of the renormalization group.

The book is addressed to graduate and Ph. D. students, to post-docs and interested colleagues, primarily from the field of particle physics. However, to a certain extent it should be of interest for colleagues from the field of statistical physics in general; some tools we are using are not standard ones from statistical physics, but were developed for solving problems in particle physics. Such tools have, for example, applications to the phase structure of spin glasses.

The style of the chapters varies from self-contained, detailed introductions, assuming basic knowledge from undergraduate lectures, to reviews of advanced research. In such reviews we explain the main ideas whose realizations are sometimes quite demanding in the techniques they use. To go there into detail would go beyond the scope of this book. Sections on perturbative approaches can be followed in detail if the reader has expertise in perturbative calculations. In general, we summarize the main ingredients in all chapters to make them self-contained, but the presentation is sometimes condensed so that a basic background knowledge in quantum field theory will facilitate the understanding. On the basis of this book it should be possible to follow the more recent literature on topics in this field.

The aim of the book is neither to give a complete or updated review on possible approaches nor a summary of the state of art nor a complete list of results for the phase structure of particle physics that are available today. The emphasis is on *principles*.

We are very much indebted to World Scientific for its patience with the submission of our manuscript. For one of us (H.M.-O.) it is also a pleasure to thank the International University Bremen, where it became possible

to finalize the book during the last years, and in particular to her student Tiberiu Tesileanu for his support in providing a camera-ready manuscript.

Bremen, July 2006

Hildegard Meyer-Ortmanns

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