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Vol. 3

COOPERATIVE EXTENSIONS OF THE BAYESIAN GAME

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*To Barbara, Yukiko, Wanda,
and the memory of LeRoy*

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Preface

Our collaboration on the research area, strategic cooperative game theory with asymmetric information, started around the spring of 2000, and progressed substantially from July 2001 through September 2002 when Ichiishi was visiting Hitotsubashi University (Yamazaki's affiliation). Through innumerable discussions during this period not only on our ongoing joint research topics but also on varieties of the literature in the area, we gradually formed and solidified our own common view about details of this theory, and at the same time developed the desire to present it systematically. The present monograph is an outgrowth of these initial efforts. We were eventually placed in an ideal environment for our collaboration as Ichiishi joined the Faculty of Hitotsubashi University in April 2005.

We have also "tried out" our results and our philosophy at: Decentralization Conference (held at Waseda University, October 6, 2001); Symposium of the Research Center for Mathematical Economics (held at Kyoto University, December 7-9, 2001); The Spring 2002 Meeting of the Japanese Economic Association (held at Otaru University of Commerce, June 15, 2002); The First Illinois Economic Theory Workshop (University of Illinois at Urbana-Champaign, April 18-20, 2003); and also at invited seminars at Kobe University, Tokyo Institute of Technology, and Kyoto University, in addition to our own seminar at Hitotsubashi University. To all the participants in our talks, we would like to express our gratitude. We would also like to thank World Scientific Co. Ltd. for publishing this monograph in this beautiful book format, in particular for their patience with our very slow writing.

Tatsuro Ichiishi and Akira Yamazaki
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