Geometry, Language and Strategy

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Geometry, Language and Strategy

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Foreword

This is a most unusual book, breaking new ground in the theory of games.

Thomas begins with the ideas of classical game theory, as formulated by von Neumann and Morgenstern. He takes this approach through a turn that gives rise to a model that has many players, each with their own game parameters and a global structure that partakes of the mathematics of differential geometry. As Thomas says in his Introduction "The best borrowings are probably of a mathematical nature, since Mathematics is superb at speaking deeply without knowing what it is talking about. For purposes of considering dynamics, I believe the ideal mathematical language is geometry." It is the nature of mathematics to have multiple interpretations and applications but it requires artistry and a conceptual shift to find the patterns that match in a new field.

Once there is a multiplicity (imagine indeed a continuum) of players (strategies) in the game, then it is natural to posit a metric that measures the relative distance between two strategies. This book creates, using geometry and physical concepts, a language appropriate to the discussion of the dynamic behaviour of strategies.

The metric in its global properties and its particulars is variable. It is affected by the very strategies that it describes. The space (metric) tells the strategies how to move, while the strategies tell the space how to curve (distort). The situation is exactly analogous to that of general relativity where space tells matter how to move and matter tells space how to curve. Thomas takes this analogy seriously and develops a theory of games that is based in the mathematical and conceptual structure of

general relativity and the allied subjects of electromagnetism and gauge theory. The result is a fascinating read that will enlighten game theorists, relativists and all readers with an active curiosity about the structure of the world and the enterprise and exploration that is the making of mathematical models.

*Louis H. Kauffman
Professor of Mathematics
University of Illinois, Chicago*

Preface

The fundamental job of leaders and managers in industry, I am told, is to determine strategy. Yet leaders and managers at best practice this as an art, not a science. Given the relative importance of strategy to leading and managing, any effort to bring discipline to the subject should be welcomed in the business community. This monograph contributes to the solution to this problem in the framework of the theory of games. I suggest a dynamic extension of the theory based on the wealth of experiences we have from geometry and the physical sciences. My goal is an ambitious one: To create a language appropriate to the discussion of the dynamic behavior of strategies as they occur in the real world. It is not only about rational behavior but also about real world behaviors. The goal of this theory is to enhance our understanding of economic behaviors and make them more susceptible to quantitative analysis.

I have been strongly influenced by work done in Systems Dynamics starting with *The Limits to Growth* by the Club of Rome (1972). Systems Dynamics [Cf. Senge (1990) and the foreword by J. W. Forrester in Wolstenholme (1990)] describes behaviors of real world systems with both economic and psychological attributes including production, manufacturing, marketing, delivery, burnout, motivation and training. Using such ideas, I characterize games as consisting of complex positive and negative feedback loops that characterize both the static *Theory of Games and Economic Behavior* by Von Neumann and Morgenstern (1944) and the communication between players which lies outside the normal rules of the game.

The positive and negative feedback loops in Systems Dynamics lead to coupled differential equations which can be solved by computer. This

is a situation not unlike physics where initial insight was achieved through phenomenology and experiment. At some point however, further progress is made by ordering the phenomenology and experimental results into a theory. This is often done through analogy to existing theories. Such an ordering creates a new set of distinctions, a language. I note the analogy that Systems Dynamics Models themselves look exactly like interacting springs with various possible connections and spring constants; just another way to think of coupled differential equations. Since such systems can be modeled as “fluids” with elasticity, viscosity, thermal conductivity *etc.*, I propose to write a theory of games as such a fluid.

I have thought about these ideas for a significant period of time, during which I have profited from conversations with many people. I am particularly grateful to Jack Behrend who steadfastly maintained that the concepts in this monograph are important and might be accessible to a CEO and encouraged me to get my ideas on paper in a simple and clear fashion. I owe a debt of gratitude to Lisa Maroski and Helen Kessler who held me to the “impossible promise” of seeing this through completion. More recently, I have enjoyed fruitful discussions with Hector Sabelli and members of his “philosophy group” concerning how the ideas of this monograph might be extended. In preparing the manuscript I have enjoyed the support of the Milwaukee School of Engineering where some of the manuscript has been prepared, my daughter Liisa Thomas for her always valuable legal advice, Steve Wolfram and the Wolfram Research, Inc. and their Partnership Program for support with Mathematica® and from the editor of this series, Lou Kauffman for encouragement. Finally, I am indebted to my wife Kathleen whose patience has made it possible to contemplate and ultimately complete this project and who also has contributed to the improvement of the look of the final manuscript applying her editorial and publishing experience. The superb help and support both mentioned and unmentioned in no way shields me from blame for any faults that remain.

G. H. Thomas

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