LATTICE METHODS FOR QUANTUM CHROMODYNAMICS

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to Laurel and Mary Ann

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Preface

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Thomas DeGrand, Carleton DeTar

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