Encyclopedia and Handbook of Process Capability Indices

A Comprehensive Exposition of Quality Control Measures

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A Comprehensive Exposition of Quality Control Measures

W. L. Pearn

National Chiao Tung University, Taiwan

Samuel Kotz

George Washington University, USA

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About the Authors

W. L. Pearn is a Professor of Operations Research and Quality Assurance at the Department of Industrial Engineering and Management, National Chiao Tung University, Taiwan. He received his Ph.D. degree in operations research from University of Maryland, College Park, MD, U.S.A. He was a quality research scientist at Bell Laboratories before joining National Chiao Tung University. He is currently Editor-in-Chief of the *Quality Technology and Quantitative Management* journal, an author and co-author of over 120 refereed journal papers. Pearn's awards include Research Assign Time Award, California State University, Performance Award, Bell Laboratories, Distinguished Research Award, National Science Council, Thomas L. Saaty Prize, and Distinguished Research Award, National Chiao Tung University.

Samuel Kotz is an Emeritus Professor of Statistics at University of Maryland, College Park, MD, U.S.A. He received his Ph.D. degree from Cornell University. He is currently a senior research scholar at the School of Engineering and Applied Sciences, George Washington University, Washington D.C., U.S.A. He was the senior Editor-in-Chief of the 13-volume Encyclopedia of Statistical Sciences, (1982-2001), an author and co-author of over 200 research papers and 14 books, in particular, a Compendium on Statistical Distributions. He holds honorary Doctorates from Harbin Institute of Technology, China, University of Athens, Greece, and Bowling State University, U.S.A., and is a recipient of the Jacob Wolfowitz prize. He is a member of the Washington Academy of Sciences, Fellow of the Royal Statistical Society, Fellow of the American Statistical Association, Fellow of the Institute of Mathematical Statistics, and an elected member of the International Statistical Institute.