ADVANCED QUANTUM T H E O R Y

The title 'ADVANCED QUANTUM T H E O R Y' is displayed in a serif font. To the right of the text are four Feynman diagrams, each corresponding to a word: 'ADVANCED' has a dashed line with a dot and arrow labeled 'q'; 'QUANTUM' has a solid line with a dot and arrow labeled 'p'; 'T' has a solid line with a dot and arrow labeled 'p'; 'H' has a wavy line with a dot and arrow labeled 'q'; 'E' has a wavy line with a dot and arrow labeled 'k'; 'O' has a wavy line with a dot and arrow labeled 'k'; 'R' has a wavy line with a dot and arrow labeled 'k'; 'Y' has a wavy line with a dot and arrow labeled 'k'.

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ADVANCED QUANTUM THEORY

THIRD EDITION

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University of Arizona, USA

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ADVANCED QUANTUM THEORY (3rd Edition)

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**To my wife and daughters,
Arlene, Kari, and Lisa**

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Preface to the Second Edition

For the past five years, my editor at Springer-Verlag has asked me to write a second edition of this text that would incorporate new material on the quark model. Because this is a subject at the forefront of modern physics, whose central ideas are perpetually in flux, such an addition is not a simple task. Nevertheless, I have tried to discuss quark model topics that should stand the test of time and be of interest to introductory advanced quantum mechanics students as examples of the Feynman diagram technique. I have also tried to eliminate errors made in the first edition.

I appreciate the work of R. Miller, who graciously typed the additional material. My colleagues V. Elias, T. Hakioglu, S. Kocić, N. Paver, and R. Thews helped me formulate the quark model chapter.

*Tucson, Arizona
May 1990*

M. D. Scadron

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Preface to the First Edition

The fundamental goal of physics is an understanding of the forces of nature in their simplest and most general terms. Yet the scientific method inadvertently steers us away from that course by requiring an ever finer subdivision of the problem into constituent components, so that the overall objective is often obscured, even to the experts. The situation is most frustrating and acute for today's graduate students, who must try to absorb as much general knowledge as is possible and also try to digest only a small fraction of the ever increasing morass of observational data or detailed theories to write a dissertation.

This book is based on the premise that to study a subject in depth is only half the battle; the remaining struggle is to put the pieces together in a broad but comprehensive manner.

Accordingly, the primary purpose of this text is to cut across the barriers existing between the various fields of modern physics (elementary particles; nuclear, atomic, and solid state physics; gravitation) and present a unified description of the quantum nature of forces encountered in each field at the level of the second-year physics graduate student. This unification is based on one-body perturbation techniques, covariantly generalized to what are now called "Feynman diagrams," and is formulated as a simple (but nontrivial) extension of ordinary nonrelativistic, one-particle quantum theory. No attempt will be made, however, to cover all or even the major topics in each field. Only those subjects will be discussed which, in the author's opinion, serve best to illustrate the utility of the Feynman-diagram approach and have a direct bearing on the fundamental forces.

In presenting the two themes of scattering diagrams and the fundamental forces, this book is divided into three parts. Part I is devoted to transformation theory, which, while admittedly somewhat formal, is the most streamlined way to review the concepts of nonrelativistic quantum mechanics and to formulate the relativistic Klein-Gordon, Maxwell, and Dirac wave equations for relativistic spin-0, massless spin-1, and spin- $\frac{1}{2}$ particles, respectively. The language of group theory is used to write relativistic Lorentz transformations in a form similar to ordinary rotations and to describe the important discrete symmetries of C , P , and T . In Part II, quantum mechanics is reformulated in the language of scattering theory, with the momentum-space S -matrix replacing the coordinate-space hamiltonian as the central dynamical operator. Nonrelativistic perturbation scattering diagrams are then developed, and simple applications given for nuclear, atomic, and solid-state scattering problems. Part III deals with relativistic scattering diagrams built up from covariant Feynman propagators and vertices in a manner consistent with the CPT theorem. The theory is systematically applied to the lowest-order fundamental electromagnetic, strong, weak, and gravitational interactions. Finally, the last chapter surveys the use of higher-order Feynman diagrams to explain more detailed aspects of quantum electrodynamics (QED) and strong-interaction elementary-particle physics. Throughout the text the notion of currents is used to exploit the underlying symmetries and dynamical interactions of the various quantum forces.

The material in Parts I and II is usually described as advanced quantum mechanics, while the topics treated in Part III often appear in texts on field theory or elementary-particle physics. It is the author's opinion, however, that every graduate student in physics should know most of the subjects covered in this book and that these topics can best be understood in proper context when presented as an integral whole. This material can be reasonably absorbed when introduced as a second-year course in quantum mechanics, with problems such as those given at the conclusion of this book thoroughly integrated into the dialogue.

It is intended that this survey of advanced, one-particle scattering quantum mechanics and Feynman diagrams will provide adequate preparation for further studies in relativistic quantum field theory or nonrelativistic many-body theory. For readers with no intention of pursuing modern quantum theory in any greater depth, it is hoped that a mastery of the material presented in this book will give them an intuitive feel for the meaning and power of the method as well as an understanding of the quantum nature of forces. At the very least, serious readers should be able to calculate simple diagrams, lifetimes, and cross sections correctly—because, according to the teaching of a famous physicist, “If you do not get the factors of 2π straight, you do not understand anything.”

The author wishes to acknowledge the kind indulgence of students in past years at the University of Arizona and Northwestern University, where much of this material was presented in somewhat unpolished form. In particular, D. Jones, T. MacMullen, B. Wilde, J. Mich, and D. Eimerol have

detected many misprints in the preliminary notes. Moreover, this work could not have been satisfactorily completed without the careful reading of the entire manuscript by Professor R. Delbourgo, much of it by Dr. H. F. Jones and various chapters by Drs. P. McNamee, P. Herczeg, C. Isham, S. Coon, K. Stelle, R. Thews, R. Young, and Professors J. D. Garcia, T. Kibble, M. Moravcsik, R. Parmenter, and P. Winternitz, all of whom offered many useful suggestions and improvements. Last but not least the author is grateful for the excellent typing and drafting support given by Barbara Bickel, Ann Williams, and Diane Hartman, the latter two of the University of Arizona, Sarah Cowen of Imperial College and W. Beiglböck and the staff at Springer-Verlag for the outstanding technical skill and speed with which they transformed the manuscript into the reality of this book.

Tucson, Arizona
August 1979

M. D. Scadron

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