Global Transversality, Resonance and Chaotic Dynamics

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Global Transversality, Resonance and Chaotic Dynamics

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Preface

The modern theory of dynamical systems originates from the Poincaré's qualitative analysis, which focuses on the complexity and stability of motions in such dynamical systems. In 1892, Poincaré discovered that the motion of nonlinear coupled oscillators is sensitive to the initial condition and qualitatively presented that the inherent characteristics of the motion in vicinity of unstable fixed points of nonlinear oscillation systems may be *stochastic* under regular applied forces. This is because the separatrix exists in nonlinear dynamical systems. Such a separatrix connected with hyperbolic points is generic, which is called the generic separatrix. However, another kind of separatrix is generated by the nonlinear resonance between the system and periodic forcing or between the two oscillation interactions. Such a separatrix is called the resonant separatrix. The natural frequency of the separatrix connected with hyperbolic points is zero and the natural frequencies of motion on both sides of the separatrix are different, which makes such a stochastic motion exists in the neighborhood of separatrix. This book will discuss the global transversality of a flow to the separatrix from a domain to another domain in order to understand the mechanism for the onset, growth and destruction of chaos in vicinity of separatrix. The author would like to present a different point of view in order to look into a fundamental theory on global transversality, resonance and chaotic dynamics in nonlinear dynamic systems. The ideas presented in this book are less formal and rigorous in an informal and lively manner. The author hopes the initial ideas may give some inspirations in the field of nonlinear dynamics.

To measure the complex behaviors in a nonlinear dynamical system, the corresponding well-behaved dynamical system is employed. For doing so, the differential geometrical relations of two flows in the two nonlinear dynamical systems are presented. Based on such differential geometric relations, this book presents a theory of global transversality, resonance and chaos in n -dimensional nonlinear dynamics. The history and recent development of nonlinear dynamics is briefly discussed first, and then the global transversality of a flow to the separatrix is investigated to determine motion complexity in n -dimensional dynamical systems. The resonant mechanism of chaos in n -dimensional dynamical systems is discussed in general. Further, the resonant theory of the stochastic layer in 2-dimensional dynamical systems is presented. In addition, the stochasticity of the resonant separatrix layers for $2n$ -dimensional, nonlinear Hamiltonian

systems is also presented, and nonlinear dynamics on a $(2n-1)$ -dimensional equi-energy surface is briefly discussed. For dissipative, nonlinear dynamical systems, the stability and grazing bifurcation are addressed. The global dynamics of 2-dimensional dissipative dynamical systems is presented. Finally, the switchability of a flow from a domain to its adjacent domain in discontinuous dynamical systems is discussed. The objective of this book is to throw out some original ideas on global transversality, resonant dynamics and chaos in nonlinear dynamics. The author believes that some ideas may not be very mature and some typos may exist in the book. The author sincerely hopes that readers can forgive such unavoidable errors here and where in the book. The author really appreciates readers for providing suggestions and comments to improve the theory presented in this book.

This book is dedicated to people who are challenging difficult and unsolved problems in natural science. Finally, I also dedicate this book to my wife (Sherry X. Huang) for support and to my lovely children (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their happiness to stimulate my inspiration.

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Contents

Preface	vii
Chapter 1. Introduction	1
1.1. A brief history of dynamics	2
1.2. Nonlinear Hamiltonian systems	5
1.2.1. Separatrix splitting	6
1.2.2. Standard and whisker maps	9
1.2.3. Chirikov resonance overlap criterion	12
1.2.4. Renormalization group technique	13
1.3. Dissipative nonlinear systems	15
1.4. Book layout	18
Chapter 2. Differential Geometry of Flows	23
2.1. Normal distance and G -functions	23
2.2. Non-contact flows	31
2.3. Contact flows	47
2.4. Concluding remarks	61
Chapter 3. Global Transversality in Continuous Dynamical Systems	63
3.1. Nonlinear dynamical systems	63
3.2. Local and global flows	66
3.3. Global transversality	71
3.4. Global tangency	80
3.5. Perturbed Hamiltonian systems	88
3.6. Two-dimensional Hamiltonian systems	92
3.7. A damped Duffing oscillator	95
3.7.1. Transversal and tangential conditions	97
3.7.2. Global transversal and tangential flows	101
3.8. Global transversality to a generalized separatrix	104
Chapter 4. Chaotic Layer Dynamics	113
4.1. Chaotic domains in phase space	113
4.1.1. Maximum and minimum first integral surfaces	114
4.1.2. First integral quantity intervals	119

4.1.3. Chaotic motion bands	125
4.2. First integral quantity increments	127
4.3. Resonance mechanism of chaotic layers	140
4.4. Energy increments in perturbed Hamiltonian systems	148
Chapter 5. Two-Dimensional Stochastic Layers	153
5.1. Geometric description in phase space	153
5.2. Approximate predictions	158
5.2.1. An incremental energy method	158
5.2.2. An accurate standard mapping technique	163
5.2.3. Energy spectrums and layer width	164
5.3. Stochastic layer in a Duffing oscillator	167
5.3.1. Incremental energy technique	170
5.3.2. Accurate standard-mapping approach	171
5.3.3. Approximate standard-mapping approach	173
5.3.4. Energy spectrum and layer width	175
5.3.5. Analytical predictions comparison	178
5.3.6. Illustrations of stochastic layers	186
5.4. Conclusions and discussions	193
Chapter 6. Stochasticity in Resonant Separatrix Layers	197
6.1. Two-dimensional resonant separatrix layers	197
6.1.1. Layer dynamics	199
6.1.2. Approximate predictions	210
6.1.2a. Onset conditions	211
6.1.2b. Vanishing conditions	214
6.1.2c. Energy increment spectrum	215
6.2. $2n$ -dimensional resonant separatrix layers	216
6.2.1. Internal resonant layer dynamics	218
6.2.2. External resonant layer dynamics	220
6.2.3. Approximate criteria	221
6.3. Resonant layers in a Duffing oscillator	222
6.3.1. Resonant layer dynamics	223
6.3.2. Analytical conditions	227
6.3.3. Numerical predictions	230
6.3.4. Illustrations of resonant layers	234
6.4. Resonant layers in a parametric pendulum	237
6.4.1. Librational resonant layer	238
6.4.2. Rotational resonant layer	243

6.4.3. Energy increment spectrum	245
6.4.4. Analytical predictions with comparison	247
Chapter 7. Nonlinear Dynamics on an Equi-energy Surface	253
7.1. Hamiltonian systems	253
7.2. Nonlinear resonance	255
7.3. Energy spectrum	258
7.4. Chaotic motions on an equi-energy surface	261
7.4.1. Resonance and averaging of Hamiltonians	263
7.4.2. Energy exchanges and criterion	265
7.4.3. Energy spectrum computation	268
7.4.4. Numerical simulations	278
7.5. Conclusions	288
Chapter 8. Stability and Grazing in Dissipative Systems	289
8.1. Equilibrium stability	289
8.1.1. System dissipativeness and expansiveness	290
8.1.2. Stability definitions	293
8.1.3. Stability conditions	295
8.2. Periodic flow stability	302
8.2.1. Stability conditions	303
8.2.2. Limit cycles	311
8.3. Local grazing bifurcation	314
8.3.1. Local grazing	315
8.3.2. Grazing mapping	316
8.3.3. Grazing determination	321
8.4. Global grazing bifurcation	324
8.4.1. Global grazing and singular sets	325
8.4.2. Global grazing conditions	326
8.4.3. Global strange attractor fragmentation	337
Chapter 9. Global Dynamics in Two-dimensional Dynamical Systems	341
9.1. Tangency and transversality	341
9.2. Energy increment and Melnikov function	343
9.3. Mapping structures	348
9.4. Bifurcation scenario	356
9.5. Numerical illustrations	362
9.5.1. Periodic flows	362
9.5.2. Grazing periodic flows	369

9.5.3. Poincarè mapping sections of chaos	374
9.5.4. Global transversality of chaos	375
9.6. Conclusions	378
Chapter 10. Flow Switchability in Discontinuous Dynamical Systems	379
10.1. Discontinuous dynamical systems	379
10.2. Passable flows	382
10.3. Non-passable flows	390
10.4. Tangential flows	400
10.5. Flow switching bifurcations	413
10.6. First integral quantity increment	434
References	437
Subject Index	445