GLOBAL DERIVATIVES

Products, Theory and Practice

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Editor

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PREFACE

The innovation in financial derivatives over the past 20 years is historically unprecedented in its scope and speed. Callable spread options, constant proportion portfolio insurance (CPPI), target redemption notes (TARN), callable snowballs, thunderballs, collateralized debt obligation (CDO) squared, himalaya, podium, galaxy, just to name a few among the more exotic products have now become ordinary. Along the same lines, new accounting standards have urged for a better comprehension of the use and risk management of financial derivatives. And the increase of computing power has also pushed to price more accurately financial derivatives with more advanced mathematical and computational tools.

In *Global Derivatives: Products, Theory and Practice*, the authors aim to provide a comprehensive introduction to the subject from a practitioner point of view with only the minimum necessary quantitative developments. All the authors are experienced financial engineers. Most of them are working for Pricing Partners and have practised cutting edge and innovative technological developments, in order to remain in “best practice”. Much has already been written on derivatives in many text books. What differentiates *Global Derivatives* from the other books on the subject is that it provides details on products, models and pricing tricks that are used in banks and often not mentioned in the academic books.

This book is designed as a graduate textbook for financial engineering course. Student should find a practitioner point of view on

the subject that could be very useful when considering a carrier in derivatives either as a quant, a trader or a structurer. This book should also appeal to practitioners, to the end users of derivatives, that is, corporate treasurers, portfolio managers, hedge funds concerned by hedging financial risks or looking for investment strategies that fit their risk appetite in a given market environment such as low interest rates and low volatility.

The first two chapters are a gentle introduction to derivatives products and markets. It explains the vanilla products such as forwards, futures, standard call and put options and their use in hedging risk. The basic option strategies are also considered.

The third chapter is about the philosophy of modeling with consideration about pricing and hedging fundamentals for fixed income products, and specific considerations for exotic and hybrid structures. In particular, Chapter 3 tries to give an insight into the models people use to price or hedge and why they make this choice. It emphasizes that the quality of a model should not only be measured in terms of pricing but also hedging. In addition, it is stressed that calibration is the art of pricing and should be closely reviewed in the pricing methodology. Before discussing more advanced pricing models, Chapter 4 provides a presentation to the seminal Black–Scholes model and its underlying hedging strategy.

Then in Chapter 5, the reader will find a description of the foundation of the fixed income derivatives market, with a description on curve bootstrapping techniques, LIBOR-in-arrears swaps, constant maturity swaps (CMS) with replication method, interest rate swaps, cross currency swaps as well as vanilla interest rate options such as caps and floors and European swaptions. Leveraging on these simple products, Chapter 6 discusses the most common models to handle the smile, that is, jump models, local volatility models and stochastic volatility models, in particular the SABR model that we are using extensively in investment banks.

Chapter 7 reviews the classic term structure models with the short-term rate models (Vasicek, Hull and White, CIR and Black and Karasinski) and the market models, especially the LIBOR market model of Brace, Gatarek and Musiela (BGM) which is very popular

among practitioners given its ability to reproduce market prices for caps, floors or swaptions when using Black's formula.

Chapter 8 is a detailed introduction to the growing market of inflation derivatives. It gives a description on the main street inflation structures such as year-on-year swaps, inflation bonds and zero-coupon swaps, and the recent market for caps and floors and swaptions. This chapter also gives a detailed account of pricing issues, such as the incorporation of seasonality, which are based on the original work of some of the authors of this book.

Chapter 9 tackles the rising and promising sector of hybrids. In particular, the reader will find a long discussion about power reverse dual currency (PRDC) that have been very popular in Japan. Chapter 10 provides an extensive overview of the latest innovative financial derivatives like callable snowballs and target redemption notes (TARN).

The book finishes with a chapter (Chapter 11) on the latest technologies developed by Pricing Partners. As the key to success in the derivatives business is financial innovation, it has become very relevant to build generic system where the end user can easily develop customized products and the corresponding pricing tools. This chapter describes the type of architecture which allows this flexibility, referred to as generic pricer. More precisely, a generic pricer is a tool that allows the description of any payoff without any new programming development. Once the payoff has been specified, the system assembles the algorithm to solve the pricing model. This is a very powerful architecture that can generate a first mover advantage to the most innovative institutions.

Last but not the least, in the appendix sections, the reader will first find a technical review of stochastic calculus and risk neutral pricing which is central to the pricing and hedging of derivatives transactions. While Appendix A is a rigorous presentation of the fundamentals of financial mathematics, it remains synthetic and does not get lost in unnecessary mathematical details that obscure the understanding of the basic concepts. The copula theory to model multivariate dependence structure between variables is also presented. This approach has become popular in finance to price "correlation

products”, that is, derivative structures whose underlying is a basket of assets such as first-to-default credit derivatives or single tranche CLOs. A detailed section on model calibration issues and the associated linear and non-linear optimization techniques complete the chapter. Appendix B provides a large review of the Monte Carlo simulation techniques with a detailed presentation of variance reduction techniques, the extension to the pricing of American Monte Carlo options and practical issues related to the derivation of the Greeks. Appendix C describes tree-based and partial differential equations (PDEs) methods, which are of great use for Bermudan and American options.

To the reader: Learn and enjoy.

Bernard Lapeyre
Ecole Nationale des Ponts
et Chaussées

ABOUT THE AUTHORS

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