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MATHEMATICS AND COMPUTATION IN IMAGING SCIENCE AND INFORMATION PROCESSING

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FOREWORD

The Institute for Mathematical Sciences at the National University of Singapore was established on 1 July 2000. Its mission is to foster mathematical research, particularly multidisciplinary research that links mathematics to other disciplines, to nurture the growth of mathematical expertise among research scientists, to train talent for research in the mathematical sciences, and to provide a platform for interaction and collaboration between local and foreign mathematical scientists, in support of national development.

The Institute organizes thematic programs which last from one month to six months. The theme or themes of a program will generally be of a multidisciplinary nature, chosen from areas at the forefront of current research in the mathematical sciences and their applications, and in accordance with the scientific interests and technological needs in Singapore.

Generally, for each program there will be tutorial lectures on background material followed by workshops at the research level. Notes on these lectures are usually made available to the participants for their immediate benefit during the program. The main objective of the Institute's Lecture Notes Series is to bring these lectures to a wider audience. Occasionally, the Series may also include the proceedings of workshops and expository lectures organized by the Institute.

The World Scientific Publishing Company has kindly agreed to publish the Lecture Notes Series. This Volume, "Mathematics and Computation in Imaging Science and Information Processing", is the eleventh of this Series. We hope that through the regular publication of these lecture notes the Institute will achieve, in part, its objective of promoting research in the mathematical sciences and their applications.

April 2007

Louis H. Y. Chen
Ka Hin Leung
Series Editors

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PREFACE

Rapid advances in communication, sensing and computational power have led to an explosion of data. The usefulness of this resource for human knowledge is determined by its accessibility and portability, which present fresh challenges to existing techniques in terms of transmission, storage, querying, display and numerical manipulation. As a result, much current research efforts focus on developing more advanced techniques for the representation, processing, analysis and interpretation of these data sets. This requires and gives rise to new theories and new methods in the areas of approximation, imaging science, information processing, mathematical modeling, scientific computing and statistics.

In view of these exciting developments, the program “Mathematics and Computation in Imaging Science and Information Processing” was held in Singapore at the Institute for Mathematical Sciences, National University of Singapore, from July to December 2003 and in August 2004 to promote multidisciplinary research on the mathematics in imaging science and information processing. In particular, the program emphasized on numerical methods in image and information processing, wavelet theory and its applications in image and signal processing, and time-frequency analysis and applications. Under the program, three conferences, six workshops, eleven tutorials and three public lectures were organized. A total of more than 340 participants took part in these activities including over 130 international attendees. We thank the Institute for Mathematical Sciences for its generous funding and efficient administrative support, without which the smooth implementation of the program would not be possible. We would also like to express our sincere appreciation to the authors for their contributions towards this volume.

The tutorials of the program, each comprising a series of lectures, were conducted by international experts, and they covered a wide spectrum of topics in the field of mathematical image, signal and information processing. This compiled volume contains survey articles by the tutorial speakers

on subdivision in geometric modeling and computer graphics, high order numerical methods for time dependent Hamilton-Jacobi equations, variational methods in mathematical image processing, data hiding and image steganography, and the apriori algorithm in data mining. The accompanying volume is on Gabor analysis and wavelet theory, which are two fundamental mathematical tools in imaging science and information processing. It contains exposition articles by the tutorial speakers and also research papers. The two volumes collectively provide graduate students and researchers new to the field a comprehensive introduction to a number of important topics in mathematical image, signal and information processing. The chapters in each volume were written by specialists in their respective areas. The following outline the organization of this volume and highlight the topics presented.

During the last decade, subdivision surfaces based on arbitrary control meshes have become an important and quite pervasive tool for computer graphics and geometric modeling applications. Chapter 1 by D. Zorin provides an introduction to the algorithms and theory related to subdivision surfaces, covering both fundamentals and recent research developments. It surveys the basic concepts on subdivision surfaces and gives an overview of various important subdivision schemes for surfaces on arbitrary meshes, with focus on two of the most common schemes (Loop and Catmull-Clark). In addition, it reviews recent theoretical results on smoothness and approximation properties of subdivision surfaces on arbitrary meshes.

Hamilton-Jacobi equations have applications in many areas including image processing and computer vision. C.-W. Shu's Chapter 2 reviews several high order numerical methods for solving time dependent Hamilton-Jacobi equations. It begins with first order monotone schemes on structured rectangular meshes and unstructured meshes, which act as building blocks for high order schemes. The high order methods discussed include essentially non-oscillatory schemes for structured meshes, weighted essentially non-oscillatory schemes for both structured and unstructured meshes, and discontinuous Galerkin schemes for unstructured meshes.

Image deblurring is the recovery of a sharp image from its blurry observation. It appears in many different sectors of imaging science, including optical, medical and astronomical applications, and is a topic of active research in mathematical image processing. Regularization techniques are often used to handle this ill-posed inverse problem. Chapter 3, co-authored by T. F. Chan and J. Shen, presents a comprehensive account of image de-

blurring, highlighting its main modeling ideas and techniques. In particular, it describes various variational methods for image deblurring for both the situations of known and unknown point spread functions which model the blurs. Mathematical analysis is provided for the existence or uniqueness of solutions of these methods. The associated computational approaches are also developed.

With the increasing need to conceal information (for instance, secret data, copyright information and movie subtitles) within a host data set, data hiding is now a major research area in signal, image and video processing. It can be regarded as a game between the embedder/decoder and the attacker, who employ optimal data-hiding and attack strategies respectively. Chapter 4 by P. Moulin and R. Koetter reviews the fundamentals of the data-hiding problem. Various data-hiding algorithms are described, ranging from simple early codes to more modern codes based on information-theoretic binning concepts. The performance of these codes are analyzed in terms of probability of error and data-hiding capacity. As illustration of the theory, image watermarking examples are shown.

Related to data hiding is the topic of steganography, in which many new and powerful techniques have been developed in the last few years. Unlike cryptography which aims to make a message secure, the goal of steganographic techniques is to hide the presence of the message itself from an observer. Due to their high degree of redundancy present, digital images are common objects used as carriers of embedded messages. Chapter 5, co-authored by M. Kharrazi, H. T. Sencar and N. Memon, is a tutorial on image steganography and steganalysis. It first introduces some general concepts and ideas in the topic of steganography, with discussions on steganographic security and capacity. Then it focuses on image steganography and steganalysis, reviewing recent techniques for embedding messages and detecting presence of messages in images.

Finally, M. Hegland's Chapter 6 is on the field of data mining. Large amount of data sets are generated by day-to-day management in various sectors such as business, finance, administration and social services. Originated from market basket analysis, association rules are currently one of the most popular tools in data mining which is about extracting useful information from these data sets. Chapter 6 gives a tutorial on the apriori algorithm for efficient association rule discovery. It begins with fundamentals of association rule discovery, and the mathematical model derived provides a framework for the apriori algorithm. Then the apriori algorithm and sev-

eral of its extensions are discussed in detail, giving much insight into this important approach of data mining.

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