International Review of Nuclear Physics Vol. 9

Hadronic Physics **from Lattice QCD**

INTERNATIONAL REVIEW OF NUCLEAR PHYSICS

Editors: T T S Kuo (*Stony Brook*) & E Osnes (*Oslo*)

Associate Editors: J Blomqvist (*Stockholm*), A Covello (*Napoli*), J Flores (*Mexico*), B Giraud (*Saclay*), A M Green (*Helsinki*), R Lemmer (*Johannesburg*), K F Liu (*Kentucky*), E Oset (*Valladolid*), F Petrovich (*Tallahassee*), J Rasmussen (*Berkeley*), J Speth (*Jülich*), D Strottman (*Los Alamos*), R Tamagaki (*Kyoto*), Y C Tang (*Minneapolis*), J Vergados (*Ioannina*)

- Vol. 1: QUARKS AND NUCLEI
edited by W Weise
- Vol. 2: COLLECTIVE PHENOMENA IN ATOMIC NUCLEI
Proceedings of the Nordic Winter School on Nuclear Physics
edited by T Engeland, J Rekstad & J S Vaagen
- Vol. 3: FEW-BODY PROBLEMS
by E Hadjimichael & W Oelert
- Vol. 4: CLUSTER MODELS AND OTHER TOPICS
by Y Akaishi, S A Chin, H Horiuchi & K Ikeda
- Vols. 5 & 6: RELATIVISTIC HEAVY ION PHYSICS
edited by L P Csernai & D D Strottman
- Vol. 7: ELECTRIC & MAGNETIC GIANT RESONANCES IN NUCLEI
edited by J Speth
- Vol. 8: NUCLEAR METHODS AND THE NUCLEAR EQUATION OF STATE
by M Baldo
- Vol. 9: HADRONIC PHYSICS FROM LATTICE QCD
edited by Anthony M Green

International Review of Nuclear Physics Vol. 9

Hadronic Physics **from Lattice QCD**

edited by

Anthony M. Green

Helsinki Institute of Physics, Finland

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401–402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

HADRONIC PHYSICS FROM LATTICE QCD

Copyright © 2004 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 981-256-022-X

Preface

The aim of this volume is to provide an introduction to Lattice Quantum Chromodynamics (LQCD) for readers that are non-specialists in the field. In particular, we have in mind advanced undergraduates and graduate students — both theorists and experimentalists — who have a background in nuclear and/or particle physics. This leads to a very limited selection being made from the many topics that are covered by LQCD. The particular topics chosen have, in most cases, analogies in more conventional nuclear and particle physics — good examples being the chapters on the interquark potential and the interactions between hadrons, both of which can be compared with the nucleon-nucleon potential and the interactions between nucleon clusters.

Why should a nuclear/particle physicist ever be interested in LQCD? This is certainly a valid question for someone with little or no knowledge of the field. For why should such a person spend their time reading about some subject that could be dead within a few years? However, in the case of LQCD, it is firmly believed that this is indeed a subject which is here to stay and, also, is expected to be continuously developed for many years to come. The reason for being so confident about this prediction can be summarised by the following phrase — “QCD is thought to be the theory of strong interactions” — words that have appeared in the literature many times over the last 30 years. Unfortunately, to prove this in its full generality has yet to be achieved. In practice, the theory can only be checked either in limiting cases — such as perturbation theory — or by some purely numerical approach — such as LQCD.

The perturbation theory approach exploits the fact that the basic QCD

interaction exhibits asymptotic freedom. This states that the strength (α) of the fundamental interaction between quarks and gluons becomes *weaker* as the momentum involved becomes *larger*. Therefore, in principle, high energy processes can be described by evaluating contributions to sufficiently high order in (α). This procedure has been applied to many cases with great success and is one of the main reasons for the above optimistic phrase about QCD being the theory of strong interactions. Unfortunately, in most situations the above perturbation limit is not applicable, since the coupling (α) is of the order unity.

In these non-perturbative regimes, attempts to deal directly with the basic QCD Lagrangian [$\mathcal{L}(\text{QCD})$] lead to singular integrals. There are ways to regulate these using, for example, cut-offs. But the outcome is an infinite number of such integrals rendering the problem as essentially still intractable. This was overcome by Wilson in 1974 through replacing $\mathcal{L}(\text{QCD})$ by $\mathcal{L}(\text{Lattice})$ — a discretized form on a 4-dimensional space-time lattice. The effect of the lattice is essentially to introduce, in a systematic manner, a high momentum cut-off of π/a , where a is the lattice spacing. This now results in a finite number of well defined integrals — but they need to be evaluated numerically. In $\mathcal{L}(\text{Lattice})$ the quark fields only exist on the lattice sites, whereas the gluon field only exists on the links between these sites. Of course, to extract observable physical quantities, the limit $a \rightarrow 0$ must eventually be taken. However, it should be emphasized that these final results are *exact* solutions of QCD — a point that was well summarised by Martin Lüscher (hep-ph/0211220) with the words: “In general numerical simulations have the reputation of being an approximate method that mainly serves to obtain qualitative information on the behaviour of complex systems. This is, however, not so in lattice QCD, where the simulations produce results that are exact (on a given lattice) up to statistical errors. The systematic uncertainties related to the non-zero lattice spacing and the finite lattice volume then still need to be investigated, but these effects are theoretically well understood and can usually be brought under control”. In most cases the size of these errors and uncertainties are indeed being continuously reduced by the access to computer resources that are ever increasing in speed and memory.

In the above, I say that there is an analogy between nuclear physics and LQCD, since both use the concept of potentials. However, it should be said immediately that this analogy can not be carried too far. For example, the

nucleon-nucleon potential $V(NN)$ plays a much more central rôle in the development of nuclear physics than the interquark potential $V(QQ)$ does in QCD. This is clearly seen by opening books on nuclear physics, where often one of the first chapters is a detailed discussion of $V(NN)$ followed by its use in, for example, the Faddeev equations for an exact description of the three nucleon systems He^3 and H^3 . Also nuclei, upto $A \approx 16$, can be understood using variational techniques. In all cases the models are expressed *directly* in terms of $V(NN)$ with all its complexities — such as tensor components and non-localities — and the results are essentially *exact*. This does not mean to say that these results agree with experiment, since there can be corrections due to three-body forces, relativistic effects, *etc.* For heavier nuclei with $A > 16$ (including nuclear and neutron matter) $V(NN)$ is first converted into an effective interaction $V_{\text{Eff}}(NN)$ by means of Brueckner-like techniques. These treat the interaction between two specific nucleons exactly but with the effects of the other nucleons only being included in an approximate way. Models for these heavier nuclei are then expressed in terms of $V_{\text{Eff}}(NN)$. The purpose of this digression is to emphasize that a major goal of nuclear physics is to explain the structure of all nuclei in terms of the basic interaction $V(NN)$. In contrast, this program is *not* generally considered to be a goal in QCD. There $V(QQ)$ is, indeed, a topic of active research, but it is not usually considered as a stepping stone in the understanding of multi-quark systems. Even for three-quark systems — the baryons — the starting point is the basic latticized lagrangian $\mathcal{L}(\text{Lattice})$ mentioned earlier. This approach treats the quarks alongwith *explicit* gluons — unlike the use of $V(NN)$ where the mesons only appear implicitly. Having just played down the rôle of $V(QQ)$ for the understanding of multi-quark systems, I should add that attempts have been made to bridge the gap between few- and multi-quark systems by way of interquark potentials. But, at present, these are still in their infancy with most of such models simply mimicking successful nuclear physics approaches.

This volume is made up from five separate chapters each of which has its counterpart in nuclear/particle physics. Also each chapter is essentially self-contained so that there is unavoidably a certain amount of repetition of, in particular, the basics of LQCD. However, this I consider to be a positive feature, since each author emphasizes different aspects that are more relevant for their particular topic.

- Chapter 1 discusses the spectroscopy of mesons and baryons — the 2 and 3 quark analogies to the deuteron and the He^3 , H^3 nuclei. Unlike nuclear/particle physics, these few-quark systems are treated directly and not in terms of $V(QQ)$. Also they exhibit a much richer spectrum, since they can be constructed from combinations of 5 different types of quark, the u , d , s , c and b quarks, whereas nuclei in general are constructed from only protons and neutrons. An exception to the latter are a few hypernuclei involving strange baryons. In contrast, there are seen charmed and bottom baryons, such as the $\Omega_c^0 = ssc$ and $\Xi_b^- = dsb$. Furthermore, these are expected to exist not only in S-wave states but also in higher partial waves — again a difference to the few-baryon systems.
- Chapter 2 discusses exotic few-quark states. In Chapter 1, the states are characterised as different combinations of the 5 types of quark with no reference to the gluon field that generates the interaction between the quarks — it being assumed to be in its ground state. In this chapter the states are, in addition, characterised by the excitation of the gluon field. This explicit manifestation of the gluon field is unavoidable in QCD, but has so far not been convincingly seen experimentally. Again this is very different to the nuclear/particle physics situation where the meson fields generating $V(NN)$, for example the π and the ρ plus their many excitations, *e.g.* the $\pi(1300)$ and the $\rho(1450)$, are well documented. In this chapter, since the term “exotic” is taken to mean states not included in the naïve quark model involving simply 2 or 3 quarks, the subject of the molecular states of two mesons is also reviewed.
- Chapter 3 discusses the basic interquark potential $V(QQ)$. This is in analogy with many text books in nuclear physics that start with the nucleon-nucleon potential $V(NN)$. Also the terminology involving spin dependence and radial forms is quite similar. However, the analogy essentially ends at that point, since — unlike the nuclear/particle physics counterpart — the study of multiquark systems generally does not build directly upon this knowledge of $V(QQ)$.
- Chapter 4 discusses the interactions between few-quark systems — the analogies to deuteron-deuteron or He^3 - He^3 scattering. Since this involves so many quarks, direct LQCD calculations, with a realistic vacuum containing quark-antiquark pairs, can not yet be

carried out using sufficiently light quarks and so, as the authors themselves say, the field is still “in an exploratory phase”. Even so it is in the direction that is necessary for bridging the gap between QCD and nuclear/particle physics.

- Chapter 5 has a different goal compared with the earlier chapters. Here the main aim is to generate lattice data that can help in the construction of models and not necessarily for direct comparison with experiment. In a sense, it is a continuation of Chapter 4 by discussing more generally the quest for bridges between QCD and nuclear/particle physics. This requires lattice calculations with multi-quark systems — mainly with 4 quarks. There is no question that models are needed for understanding multi-quark systems, where the number of quarks is much larger than four, since LQCD calculations are not feasible for general multi-quark systems. In fact, there are on-the-market many multi-quark models that are “inspired” by QCD. Unfortunately, few are actually “based” on QCD — the inspirations mainly coming from successful multi-nucleon models.

Naturally, as with most projects involving authors distributed world-wide, there have been some “communication” problems. Fortunately, these were eventually resolved but — in one case — not without considerable cajoling. Several of the contributors actually expressed their pleasure for the opportunity to review their work for non-experts. But, of course, hopefully the volume will prove most useful for the audience it is primarily intended — the non-experts.

A. M. Green

This page is intentionally left blank

Contents

Preface	v
Chapter 1 Meson and Baryon Spectroscopy on a Lattice	1
<i>C. McNeile</i>	
1.1 Introduction	1
1.2 Basic Lattice Gauge Theory	4
1.3 Systematic Errors	16
1.3.1 Unquenching	17
1.3.2 Lattice spacing errors	19
1.3.3 Quark mass dependence	22
1.3.4 Finite size effects	39
1.4 An Analysis of Some Lattice Data	42
1.5 Parameter Values of Lattice QCD Calculations	45
1.6 The Masses of Light Mesons	49
1.6.1 P-wave mesons and higher excitations	54
1.7 The Masses of Light Baryons	59
1.7.1 Excited baryon states	68
1.8 Electromagnetic Effects	72
1.9 Insight from Lattice QCD Calculations	75
1.10 What Lattice QCD Is Not Good at	81
1.11 Conclusions	86
Acknowledgements	87
Appendix: Technical Details	88
Bibliography	90

Chapter 2 Exotics	103
<i>C. Michael</i>	
2.1 Introduction	103
2.2 Glueballs and Scalar Mesons	106
2.2.1 Glueballs in quenched QCD	106
2.2.2 Scalar mesons in quenched QCD	109
2.2.3 Scalar mesons in full QCD	110
2.2.4 Experimental evidence for scalar mesons	111
2.3 Hybrid Mesons	112
2.3.1 Heavy quark hybrid mesons	113
2.3.2 Hybrid meson decays	117
2.3.3 Light quark hybrid mesons	118
2.4 Hadronic Molecules	120
2.5 Conclusions and Outlook	123
Acknowledgements	124
Bibliography	124
 Chapter 3 Two Quark Potentials	 127
<i>G. Bali</i>	
3.1 Motivation	127
3.2 The Static QCD Potential	129
3.2.1 Wilson loops	130
3.2.2 Exact results	136
3.2.3 Strong coupling expansions	137
3.2.4 String picture	140
3.2.5 The potential in perturbation theory	142
3.3 Quark-Antiquark Potentials between Non-Static Quarks	146
3.3.1 Radial form of $V(Q\bar{Q})$	147
3.3.2 Comparison with the form of $V(NN)$	150
3.4 Conclusions	151
Acknowledgements	152
Bibliography	152

Chapter 4 Interactions between Lattice Hadrons	157
<i>H.R. Fiebig and H. Markum</i>	
4.1 Introductory Overview on Goals, Strategies, Methods	158
4.1.1 Modeling nuclear forces	158
4.1.2 The lattice QCD perspective	162
4.1.3 Short and long term goals for lattice QCD	163
4.1.4 Probing the lattice	164
4.1.5 Finite-size methods	166
4.1.6 Residual interaction	167
4.1.7 Use of improved actions	169
4.2 A Simple U(1) Lattice Model in 2+1 Dimensions	170
4.2.1 Lattice action	170
4.2.2 Meson fields	171
4.2.3 Correlation matrices	172
4.2.4 Correlation matrix for noninteracting mesons	175
4.2.5 Computation with random sources	177
4.3 Effective Residual Interaction	180
4.3.1 Perturbative definition	180
4.3.2 Effective interaction for composite operators	185
4.3.3 Lattice symmetries	185
4.3.4 Truncated momentum basis	187
4.3.5 Adiabatic approximation	190
4.3.6 Analysis on a periodic lattice	195
4.4 Current State of QCD in 3+1 Dimensions	197
4.4.1 Scattering lengths for π and N systems	197
4.4.2 Static $N - N$ and $N - \bar{N}$ potentials	202
4.4.3 Heavy-light meson-meson systems	204
4.4.4 Momentum-space work on the $\pi - \pi$ system	214
4.4.5 Coordinate-space work on the $\pi - \pi$ system	219
4.5 Conclusion and Outlook	222
Acknowledgements	225
Appendix A: Remarks on Staggered Fields	225
Appendix B: Improved Lattice Actions	227
B.1: A scalar example	228
B.2: Improvement of a pure gauge theory	232
B.3: Improvement of a fermionic action	237
B.4: More on highly improved actions	239
B.5: Clover leaf fermion action	241
Bibliography	243

Chapter 5	Bridges from Lattice QCD to Nuclear Physics	249
	<i>A.M. Green</i>	
5.1	Introduction	249
5.1.1	Numerical treatment of QCD	250
5.1.2	Effective Field/Potential Theories	254
5.1.2.1	Effective Field Theories (EFTs)	254
5.1.2.2	Effective Potential Theories (EPTs)	257
5.2	What Is Meant by “A Bridge”?	262
5.2.1	A simple example of a bridge	263
5.2.1.1	Setting the scale from the string tension	263
5.2.1.2	Sommer’s prescription for setting the scale	264
5.2.2	Are there bridges other than $V_{Q\bar{Q}}$?	265
5.3	The Energies of Four Static Quarks ($QQ\bar{Q}\bar{Q}$)	271
5.3.1	Quark descriptions of hadron–hadron interactions	271
5.3.2	The rôle of lattice QCD	272
5.4	$(Q\bar{Q})$ and $[(Q\bar{Q})(Q\bar{Q})]$ Configurations	274
5.4.1	Lattice calculations with $(Q\bar{Q})$ configurations	275
5.4.1.1	Generating lattice configurations	276
5.4.1.2	Appropriate operators on a lattice	277
5.4.1.3	Fuzzing	280
5.4.2	Lattice calculations with $[(Q\bar{Q})(Q\bar{Q})]$ configurations	283
5.4.3	Lattice parameters and finite size/scaling check	286
5.4.3.1	Benchmark data	286
5.4.3.2	Finite size effect	286
5.4.3.3	Smaller lattice spacing	287
5.5	Potential Model Description of the Lattice Data	288
5.5.1	Unmodified two-body approach	288
5.5.2	The effect of multi-quark interactions	291
5.5.3	A compromise for the overlap factor f	294
5.5.4	The effect of two-gluon exchange	295
5.5.5	Parametrizations of the gluon-field overlap factor f	296
5.5.5.1	A reason for $f_1 = \exp[-\alpha_b S]$	298
5.6	More Complicated $[(Q\bar{Q})(Q\bar{Q})]$ Geometries	300
5.6.1	Tetrahedral configurations on a lattice	300
5.6.2	QCD in two dimensions (1+1)	305
5.7	Extensions of the 2×2 f -Model	305
5.8	Heavy-Light Mesons ($Q\bar{q}$)	307

5.8.1	Bottom (B)-mesons	308
5.8.2	Lattice parameters	309
5.8.3	Maximal Variance Reduction (MVR)	310
5.8.4	Energies of heavy-light mesons ($Q\bar{q}$)	312
5.8.4.1	Two-point correlation functions C_2	312
5.8.4.2	Analysis of C_2 to extract energies	313
5.9	Charge and Matter Distributions of Heavy-Light Mesons ($Q\bar{q}$) .	315
5.9.1	Three-point correlation functions C_3	315
5.9.2	Analysis of C_3	317
5.9.3	Fits to the radial forms	320
5.9.3.1	Fitting data with Yukawa, exponential and gaussian forms	320
5.9.3.2	Fitting $Q\bar{q}$ data with the Dirac equation . . .	321
5.9.3.3	Fitting $Q\bar{q}$ data with the Schrödinger equation	324
5.9.3.4	Fitting $Q\bar{q}$ data with semirelativistic equations	326
5.9.4	Sum rules	327
5.10	The $B - B$ System as a $[(Q\bar{q})(Q\bar{q})]$ Configuration.	329
5.10.1	Lattice calculation of the $[(Q\bar{q})(Q\bar{q})]$ system	329
5.10.2	Extension of the f -model to the $[(Q\bar{q})(Q\bar{q})]$ system . . .	333
5.11	The $B - \bar{B}$ System as a $[(Q\bar{q})(\bar{Q}q)]$ Configuration	335
5.12	Conclusions and the Future	338
	Acknowledgements	344
	Appendix A: Extensions of the f -Model from 2×2 to 6×6	345
	A.1: The 3×3 extension of the unmodified two-body approach of Subsec. 5.5.1	345
	A.2: The 3×3 extension of the f -model of Subsec. 5.5.2 . . .	346
	A.3: The 6×6 extension of the f -model of Subsec. 5.5.2 . . .	352
	Appendix B: Extension of the f -Model to $[(Q\bar{q})(Q\bar{q})]$ Systems . . .	355
	Bibliography	359