Vehicle-Bridge Interaction Dynamics

With Applications to High-Speed Railways

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Preface

The commercial operation of the first high-speed (or bullet) train in 1964 with a speed of 210 km/hr in the Japanese railways connecting Tokyo and Osaka marked the beginning of a new era in railway engineering. Since then, high-speed trains with speeds over 200 km/hr or higher have emerged as a competitive tool for inter-city transportation in several countries including Japan, Germany, France, Italy, Spain, United Kingdom and Sweden. Such a trend continues to spread in different parts of the world. While Japan and many European countries have been working on expanding their high-speed railway networks or improving their existing railway lines, Asian countries, such as Korea, Taiwan and China, have reached the stage of planning, constructing, or field-testing their high-speed railway systems. Undoubtedly, high-speed train will become a key tool for inter-city passenger transportation, at least in the aforementioned countries.

Partly enhanced by the rapid expansion of high-speed railway systems, research on the moving load problems in general, and *vehicle-bridge interactions*^a in particular, has been booming in the past two decades. Nevertheless, there is an apparent lack of a timely book that can adequately address most of the problems encountered in the design of high-speed railway bridges, which for the reasons stated

^aIn the literature, the term “bridge-vehicle interaction” was also used. It is realized that such a term was used by those who place more emphasis on the bridge than on the moving vehicles. In this text, we prefer to use the term “vehicle-bridge interaction”, since we place equal weights on the dynamic behavior of the bridge and moving vehicles.

below, are different from those encountered in traditional railway or highway bridges. This book is intended to fill such a gap. It has been developed as a result of the research works conducted by the authors and their co-workers. In preparing this book, special attention was paid to the problems that may be encountered by engineers in practice, with clear physical meanings given for each of the phenomena involved. It is hoped that the book in the present form can serve as a most updated source of reference for engineers and researchers working in high-speed railways, and possibly to those working in the broad area of railway or bridge engineering.

One problem encountered in high-speed railways is the impact and vibration of bridges caused by the moving trains. This problem is substantially different from that encountered in highway bridges for the following reasons. First, the loads induced by a moving train on the bridge are *repetitive* in nature, as characterized by the sequentially moving wheel loads, implying that certain frequencies of excitation will be imposed on the bridge during the passage of a train. In contrast, the loads implied by a highway traffic are random in nature, when expressed in terms of the wheel loads and wheel distance. Second, high-speed trains can travel at a speed much higher than the vehicles moving on highways, making it possible for the excitation frequencies to coincide with the vibration frequencies of the bridge, resulting in the so-called *resonance* phenomenon. Whenever the condition of resonance is reached, the bridge response will be continuously amplified as there are more wheel loads passing the bridge. Such a phenomenon can hardly be observed in highway bridges. Third, the *mass ratio* of the vehicles to the bridge is generally larger for railways than for highways, due to the fact that a train consists of a number of cars in connection and the railway bridge deck is relatively narrow, it carries no more than two tracks in most cases. In contrast, a highway bridge deck may be so wide that it can afford four or more lanes of running vehicles in each of the two directions. For this reason, the *interaction* between the moving vehicles and bridge appears to be much stronger for railways than for highways. Finally, concerning the maneuverability of the train in motion, the *riding comfort* or *vehicle response* is an issue that

should be taken into account in the design of high-speed railways. Moreover, the response of a moving vehicle is more sensitive to the *vehicle-bridge interaction* (VBI) compared with that of the bridge. However, the analysis of the dynamic behavior of a VBI system is not straightforward as there are two subsystems, i.e., the moving vehicles and the bridge, interacting with each other through the *contact forces* existing between the wheels and rails surface, which, in essence, is a nonlinear, coupled and time-dependent problem.

This book intends to give a broad and systematic coverage of the vibration problems encountered in high-speed railway bridges, with particular emphasis placed on the interaction between the moving vehicles and supporting bridge. In general, the book is divided into two parts, with Part I dedicated to the moving load problems and Part II to the interaction dynamics problems. These two parts can also be distinguished by the fact that the moving load problems (i.e., those treated in Part I) can generally be solved by analytical means, for which closed form solutions are possible, while the interaction dynamics problems (i.e., those treated in Part II) can only be solved by numerical means, say, using the vehicle-bridge interaction elements derived.

Starting with a general review of the related previous works in Chapter 1, an analytical formulation was presented for simply-supported beams subjected to a sequence of moving loads in Chapter 2, from which the phenomena of *resonance* and *cancellation* were identified, along with the optimal design criteria established for bridges. The closed-form solution presented for simple beams allows us to identify the key parameters involved. Conventionally, elastic bearings are installed at the supports of bridge girders for isolating the earthquake forces transmitted from the ground to the superstructure. However, such devices may adversely result in amplification of the response of the bridge during the passage of a train. The problem of elastically supported beams subjected to moving loads has received little attention in the literature, which was studied by an analytical approach in Chapter 3. The envelope impact formulas presented in Chapter 3 can be used as a useful aid for preliminary designs. Moreover, the mechanism for the occurrence of resonance and

cancellation was thoroughly investigated in Chapter 4, with which the measured results obtained from the field test for two adjacent bridges was interpreted with clear physical meanings.

The dynamic behavior of a horizontally-curved beam subjected to a series of moving masses was formulated and studied in Chapter 5. This problem was not well-treated before, due to the overlook of the centrifugal forces induced by masses moving over a circular path, which are functions of both the speed of the moving masses and radius of the curved beam. In Chapter 5, a complete theory was presented for the vertical and horizontal vibrations of a horizontally-curved beam under the excitation of the gravitational and centrifugal forces, respectively, that are induced by the moving masses. Particular emphasis was placed on the impact effect caused by the moving masses on the radial response of the curved beam.

One feature of the book is the derivation of a number of efficient VBI elements by condensing the vehicle's degrees of freedom to those of the bridge in contact, based on the concept of dynamic condensation in Chapter 6. These elements can be used to simulate problems with bridges and moving vehicles of various complexities. The VBI element presented in Chapter 6 was extended in Chapter 7 to include the pitching motion of the moving vehicle. Using the VBI elements derived, the dynamic properties of the vehicles and bridge, as well as rail irregularities, can be duly taken into account, while the dynamic response of the moving vehicle can be solved in addition to the bridge response.

Another way to analyze the VBI dynamics is to treat the moving vehicles and bridge as two separate systems, which interact with each other through the contact forces. By solving for the contact forces from the vehicles equations, one can treat them as external forces acting on the bridge, which can then be solved using conventional finite element procedures. Such a concept was utilized in developing the VBI element in Chapter 8, which was then extended in Chapter 9 to include the effect of rails with profile irregularities that form part of a railway track in the two-dimensional sense. Because of its versatility, the VBI element derived, based on the concept of contact forces, can be used in the simulation of various three-dimensional

vehicle-rails-bridge systems considering, for instance, the crossing of two trains on a bridge, the risk of derailment of a moving train (Chapter 10), and the stability of trains moving over bridges simultaneously shaken by earthquake (Chapter 11).

The authors wish to express their sincerest gratitude to their great teacher in civil engineering and education, Dr. Chao-Chung Yu, the former dean of the College of Engineering, National Taiwan University (NTU) (1972–1979) and the former President of the NTU (1981–1984), for his strong influence and continuous advice through their careers of development, both as students and teachers. His experience as a teacher, researcher, educationist, and in some sense as an engineer, has always been a source of inspiration to all the young fellows under his instruction or working with him.

A large portion of the research results presented in this book has been sponsored through a series of research projects granted by the National Science Council of the Republic of China on subjects related to vehicle-bridge interactions, as well as on bridge dynamics. The senior author has been the principal investigator of all these projects. Without such a continuous support, it would be difficult to maintain such a large research group at the NTU working on different aspects of the VBI problem, ranging from the vibration of substructure and superstructure of railway bridges to wave propagations in soils and nearby buildings; the latter forms an independent subject that requires further research, which was not covered in this book. Besides, we are also grateful to the China Engineering Consultants, Inc., for their continuous support to our research group, especially through the 1st Structural Department previously led by Senior Vice President Mr. Dyi-Wei Chang. Some research results presented in this book have been made possible through such a support.

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- Yang, Y. B., Chang, C. H., and Yau, J. D. (1999). "An element for analysing vehicle-bridge systems considering vehicle's pitching effect," *Int. J. Numer. Meth. Eng.*, **46**, 1031–1047. ©John Wiley & Sons Limited, reproduced with permission.
- Yang, Y. B., Lin, C. L., Yau, J. D., and Chang, D. W. (2004). "Mechanism of resonance and cancellation for train-induced vibrations on bridges with elastic bearings," *J. Sound & Vibr.*, **269**(1–2), 345–360. Reproduced with permission from Elsevier.

- Yang, Y. B. and Yau, J. D. (1997). “Vehicle–bridge interaction element for dynamic analysis,” *J. Struct. Eng.*, ASCE, **123**(11), 1512–1518 (Errata: **124**(4), 479). Reproduced by permission of ASCE.
- Yang, Y. B., Yau, J. D., and Hsu, L. C. (1997b). “Vibration of simple beams due to trains moving at high speeds,” *Eng. Struct.*, **19**(11), 936–944. Reproduced with permission from Elsevier.
- Yang, Y. B., Wu, C. M., and Yau, J. D. (2001). “Dynamic response of a horizontally curved beam subjected to vertical and horizontal moving loads,” *J. Sound & Vibr.*, **242**(3), 519–537. Reproduced with permission from Elsevier.
- Yang, Y. B. and Wu, Y. S. (2001). “A versatile element for analysing vehicle–bridge interaction response,” *Eng. Struct.*, **23**, 452–469. Reproduced with permission from Elsevier.
- Yang, Y. B. and Wu, Y. S. (2002). “Dynamic stability of trains moving over bridges shaken by earthquakes,” *J. Sound & Vibr.*, **258**(1), 65–94. Reproduced with permission from Elsevier.
- Yau, J. D., Wu, Y. S., and Yang, Y. B. (2001). “Impact response of bridges with elastic bearings to moving loads,” *J. Sound & Vibr.*, **248**(1), 9–30. Reproduced with permission from Elsevier.
- Yau, J. D., Yang, Y. B., and Kuo, S. R. (1999). “Impact response of high speed rail bridges and riding comfort of rail cars,” *Eng. Struct.*, **21**(9), 836–844. Reproduced with permission from Elsevier.

List of Symbols

The following is a list of symbols used throughout this book. All the symbols are defined at the place where they first appear in the text. Matrices, column and row vectors are enclosed by $[\]$, $\{ \ }$, and $\langle \ \rangle$, respectively. A quantity occurring at time t and $t + \Delta t$ are denoted with subscript t and $t + \Delta t$, respectively. A dot placed over a quantity is used to denote the derivative of the quantity with respect to time t . And a prime attached to a quantity is used to denote the derivative of the quantity with respect to coordinate x . Only quantities that are not confined to local use are listed below.

A	cross-sectional area of beam
a	acceleration of vehicle
$a_0 \sim a_7$	coefficients as defined in Eq. (B.4)
$B(f_i)$	weighting factor, Eq. (9.48)
BYQ	bogie-side lateral to vertical force ratio
$b_0 \sim b_7$	coefficients as defined in Eq. (8.8)
$[C]$	damping matrix of structure
$[C_b]$	damping matrix of bridge free of any vehicle actions
C_n	n th modal damping coefficient
$[C_0]$	damping matrix of railway bridge free of any vehicle actions
$[c_b]$	damping matrix of bridge element
$[c_{bi}]$	damping matrix of e_i th element of bridge
$[c_c]$	contact matrix as defined in Eq. (8.14)

$[c_{ij}^*]$	damping matrix caused by linking action of car body
$[c_d]$	rail damping matrix due to interaction with the bridge
c_e	external damping coefficient
c_i	internal damping coefficient
$[c_{ii}], [c_{jj}]$	damping matrices for element i and j , Eqs. (7.25) and (7.26)
$[c_{ij}], [c_{ji}]$	damping matrices related to pitching actions for element i and j , Eqs. (7.25) and (7.26)
$[c_r]$	damping matrix of rail element
$[c_{rl}]$	damping matrix of LSR element
$[c_{rr}]$	damping matrix of RSR element
c_t	material damping coefficient for translational motion of track
c_t^*	material damping coefficient for torsional motion of track
$[c_{uu}]$	a partitioned damping matrix of vehicle
$[c_{uw}]$	a partitioned damping matrix of vehicle
c_v	damping coefficient of suspension unit
$[c_v]$	damping matrix of vehicle
$[c_{wu}]$	a partitioned damping matrix of vehicle
$[c_{ww}]$	a partitioned damping matrix of vehicle
D	determinant of matrix
$\{D\}$	structural displacement vector
d	length of train car
$\{d_b\}$	displacement vector of bridge
$\{d_{bi}\}$	nodal displacement vector of e_i th element of bridge
$\{d_b^n\}$	natural deformations of bridge
$\{d_b^r\}$	rigid displacements of bridge
$\{d_c\}$	displacement vector of contact points of bridge or rail
$\{d_e\}$	displacement vector of car body
$\{d_f\}$	displacement vector of front bogie

$\{d_r\}$	displacement vector of rail element
$\{d_r\}$	displacement vector of rear bogie, Chapter 10
$\{d_{rl}\}$	displacement vector of LSR element
$\{d_{rr}\}$	displacement vector of RSR element
$\{d_u\}$	displacement vector of upper part of car body
$\{d_v\}$	displacement vector of vehicle, $\{d_v\} = \langle \{d_u\} \{d_w\} \rangle^T$
$\{d_w\}$	displacement vector of wheel part of car body
$\{d_{wi}\}$	displacement vector for i th wheel
d_{wb}	wheelbase of each wheelset
E	modulus of elasticity
$F(t)$	load function
$\{F\}$	external nodal forces of structure
$\{F_b\}$	external nodal forces of bridge
$\{\bar{F}_b\}$	effective resistant force vector of structure
$F_k(v, t)$	generalized forcing function
$\{f_A\}$	nodal loads of rail element of Track A
$\{f_A^t\}$	total equivalent nodal forces of element under earthquake
f_b	frequency of vibration of bridge unit
$\{f_b\}$	external nodal forces of bridge element
$\{f_{bci}\}$	vector of consistent nodal forces for i th contact force
$\{f_{bi}\}$	vector of external forces for e_i th element of bridge
f_c	contact force
$\{f_c\}$	vector of contact forces
$\{f_c^*\}$	general vector of contact forces
$f_{c1} \sim f_{c4}$	contact forces
$\{f_e\}$	external force components excluding contact forces
f_h	horizontal moving load
$\{f_r\}$	nodal loads of rail element
$\{f_{rci}\}$	equivalent nodal forces caused by i th vertical contact force
$\{f_{rl}\}$	nodal loads of LSR element

$\{f_{rr}\}$	nodal loads of RSR element
f_{rot}	rotational moment experienced by car body
$\{f_s\}$	resistant forces of sprung mass unit
$\{f_{ue}\}$	external forces acting on upper part of vehicle
f_v	gravitational load of moving vehicle, $f_v = -m_v g$
$\{f_v\}$	force vector of vehicle
f_{ver}	vertical force experienced by car body
$\{f_{we}\}$	external forces acting on wheel part of vehicle
G	shear modulus
g	acceleration of gravity
H	unit step function
H_i	i th horizontal contact force
h	vertical distance between deck and torsional center of cross section
h_{ci}	lateral displacement of i th contact point
I	impact factor
I	moment of inertia (used together with E)
I_M	impact factor for bending moment
I_p	polar moment of inertia of beam
I_u	impact factor for deflection
I_V	impact factor for end shear force
I_v	rotatory inertia of car body
I_y, I_z	moments of inertia of beam about y and z axes
J	torsional constant
K	stiffness of elastic bearings
$[K]$	stiffness matrix of structure
$[K_b]$	stiffness matrix of bridge structure
$[\bar{K}_b]$	effective stiffness matrix of structure
$[K_{eff}]$	effective stiffness matrix
$[K_0]$	stiffness matrix of railway bridge free of any vehicle actions
k_B	stiffness of ballast
$[k_b]$	stiffness matrix of bridge element
$[\hat{k}_b]$	stiffness matrix of condensed system
$[k_{bi}]$	stiffness matrix of e_i th element of bridge

$[k_c]$	contact matrix as defined in Eq. (8.14)
$[k_{cij}^*]$	stiffness matrix caused by linking action of car body
$[k_{ii}], [k_{jj}]$	stiffness matrices for element i and j , Eqs. (7.25) and (7.26)
$[k_{ij}], [k_{ji}]$	stiffness matrices related to pitching actions for element i and j , Eqs. (7.25) and (7.26)
$[k_r]$	stiffness matrix of rail element
$[k_{rl}]$	stiffness matrix of LSR element
$[k_{rr}]$	stiffness matrix of RSR element
$[k_s]$	rail stiffness matrix due to interaction with bridge
$[k_{uu}]$	a partitioned stiffness matrix of vehicle
$[k_{uw}]$	a partitioned stiffness matrix of vehicle
k_v	stiffness of suspension unit
$[k_v]$	stiffness matrix of vehicle
$[k_{wu}]$	a partitioned stiffness matrix of vehicle
$[k_{ww}]$	a partitioned stiffness matrix of vehicle
L	span length or characteristic length of bridge
L_c	distance between two wheel assemblies of train car
L_d	length equal to car length minus the axle distance
L_r	length of irregularity
l	length of beam element
$[l]$	transformation matrix, Chapters 8 and 9
l_a	half of axle length of wheelset
M	car mass lumped at each load
$M(x, t)$	bending moment
$[M]$	mass matrix of structure
$[M_b]$	mass matrix of bridge structure
M_c	mass of car body
M_n	n th modal mass
M_t	mass of bogie
M_v	half of the mass of car body
M_w	mass of vehicle
$[M_0]$	mass matrix of railway bridge free of any vehicle actions

m	mass per unit length
$[m_b]$	mass matrix of bridge element
$[m_{bi}]$	mass matrix of e_i th element of bridge
$[m_c]$	contact matrix as defined in Eq. (8.14)
$[m_{cij}^*]$	mass matrix caused by linking action of car body
$[m_{ii}], [m_{jj}]$	mass matrices for element i and j , Eqs. (7.25) and (7.26)
$[m_r]$	mass matrix of rail element
$[m_{rl}]$	mass matrix of LSR element
$[m_{rr}]$	mass matrix of RSR element
$[m_{uu}]$	a partitioned mass matrix of vehicle
$[m_{uw}]$	a partitioned mass matrix of vehicle
m_v	mass of moving vehicle
$[m_v]$	mass matrix of vehicle
m_w	mass of wheel assembly
$[m_{wu}]$	a partitioned mass matrix of vehicle
$[m_{ww}]$	a partitioned mass matrix of vehicle
N	total number of moving loads
$\{N_c\}$	interpolation vector for beam displacement evaluated at contact point x_c , $\{N_c\} = \{N(x_c)\}$
$\{N_{ci}^h\}$	interpolation vector of bridge element evaluated for i th horizontal contact force
$\{N_{ci}^v\}$	interpolation vector of bridge element evaluated for i th vertical contact force
N_{min}	minimum number of bridge units used in analysis
$\langle N_u \rangle$	interpolation vector for axial displacement
$\langle N_v \rangle$	interpolation vector for vertical displacement
n_A	number of train cars moving on Track A
n_B	number of train cars moving on Track B
n_b	number of CFR elements considered within minimal bridge segment
n_v	number of vehicles comprising the train
P	unified load function as defined in Eq. (5.45)
$\{P\}$	applied loads of structure
$\{P_b\}$	external loads of bridge

$\{P_c^*\}$	equivalent contact forces of structure
PD	axle load decrement ratio
$\{P_{eff}\}$	effective load vector
$P_n(x, t)$	function as defined in Eq. (2.50)
PQ	axle decrement ratio
$P_1(v, t), P_2(v, t)$	response functions as defined in Eq. (3.15)
$\bar{P}_1(v, t)$	response function as defined in Eq. (2.54)
p	load magnitude, $p = -(M_v + m_w)g$, Chapter 6
$\{p_b\}$	nodal loads of bridge element
$\{p_c\}$	load vector as defined in Eq. (8.15)
(p_c, p_k)	unit axial interaction forces between rail and bridge elements
$\{p_{ci}^*\}$	equivalent loads as defined in Eq. (8.24)
$\{p_v\}$	load vector of sprung mass model, $\{p_v\}^T = \langle p, 0 \rangle$
$\{p_w\}$	loads induced by wheels
p_{x1p}, p_{z1p}	particular solutions for in-plane vibrations of curved beam
p_{x1h}, p_{z1h}	homogeneous solutions for in-plane vibrations of curved beam
$p_{y1p}, p_{\theta 1p}$	particular solutions for out-of-plane vibrations of curved beam
$p_{y1h}, p_{\theta 1h}$	homogeneous solutions for out-of-plane vibrations of curved beam
$\{Q_c^*\}_t$	equivalent contact forces of structure
$Q_1(v, t), Q_2(v, t)$	response functions as defined in Eq. (3.21)
$\{q_c\}$	load vector as defined in Eq. (8.15)
(q_c, q_k)	unit vertical interaction forces between rail and bridge elements
$\{q_{ci}^*\}$	equivalent loads as defined in Eq. (8.24)
$q_n(t)$	n th generalized coordinate
$\{q_s\}$	internal resistant forces of sprung mass unit
$\{q_u\}$	equivalent nodal loads for upper part of car body
$\{q_{uc}\}$	equivalent contact forces for upper part of car body

q_{x1}	first generalized coordinate for axial displacement u_x
q_{yi}	i th generalized coordinate for displacement u_y
q_{zi}	i th generalized coordinate for radial displacement u_z
$q_{\theta i}$	i th generalized coordinate for angle of twist θ_x
R	radius of curvature of curved beam
R_d	maximum dynamic response
R_s	maximum static response
R_{sM}	maximum bending moment under static loads
R_{su}	maximum static deflection
R_{sV}	maximum static shear force
$r(x)$	profile of rail irregularity
$\{r\}$	vector of irregularity profile at contact points
r_c	elevation of rail irregularity at contact point x_c
(r_c, r_k)	unit vertical interaction forces between rail and bridge elements
$r_h(x)$	deviation in lateral alignment of two rails
r_i	track profile evaluated at i th contact point
$r_{v1}(x), r_{v2}(x)$	vertical deviations of two rails
r_0	amplitude of irregularities
r_0	nominal radius of wheel, Chapter 10
S	speed parameter
S_c	speed parameter for cancellation
S_{h1}	speed parameter for the horizontal vibration of curved beam
S_n	n th speed parameter
S_r	speed parameter for resonance
S_{v1}	speed parameter for vertical vibration of curved beam
SYQ	single wheel lateral to vertical force ratio
$S(\Omega)$	power spectral density (PSD) function for track irregularity
t	time

t_c	time lag between two sets of moving loads ($= L_c/v$)
t_{end}	ending time for analysis
t_j	arriving time of j th load on beam
t_N	arriving time of N th load on beam
$\{U_b\}$	total displacements of bridge
$U_{N,1}$	forced vibration caused by N th moving load, Eq. (5.51)
$U_{N,2}$	residual vibration caused by $N - 1$ moving loads, Eq. (5.52)
$U_j(t, v, L)$	load configuration as defined in Eq. (2.34)
$U_j(x, t)$	residual vibration caused by j th moving load, Eq. (5.50)
$U(x, t)$	unified displacement function as defined in Eq. (5.45)
$\{U\}$	displacements of structure
$\{U_b\}$	displacements of bridge structure
u_b	axial displacement of bridge element, Chapter 9
$\{u_b\}$	nodal displacements of beam element
$\{u_g\}$	support displacements of bridge
$u(x, t)$	deflection of beam at section x and time t
u_i, u_j	vertical displacements of elements i and j
u_r	axial displacement of rail element
u_x, u_y, u_z	cross-sectional displacements of curved beam along three axes
$V(x, t)$	shear force
V_i	vertical force acting at i th contact point of bridge
v	vehicle speed
v_b	vertical displacement of bridge element, Chapter 9
v_{ci}	vertical displacement of i th contact point of bridge
v_r	vehicle speed at resonance
v_r	vertical displacement of rail element, Chapter 9
v_u	displacement of car body mass

v_w	displacement of wheel mass
v_{wi}	displacement of i th wheel
v_0	initial velocity of vehicle
W	half weight of car body, $W = 0.5 M_v g$
W	static weight of each wheelset, Chapter 10
W_z	Sperling's ride index
W_{z_i}	comfort index
x	beam axis
x_{bs0}	position for first wheel to start acceleration or braking
x_{bsf}	position for first wheel to stop acceleration or braking
x_c	position of contact point
x_{ci}, x_{cj}	position of contact point for elements i and j
x_{end}	ending position of first wheel
x_0	reference distance in Eq. (6.38)
Y_{lim}	limit on lateral track force
y	a cross-sectional axis
y_1, y_2	coordinates of the two masses of sprung mass model, Chapter 6
$\{y\}$	nodal vector of sprung mass model, $\{y\}^T = \langle y_1, y_2 \rangle$, Chapter 6
$\{y\}$	displacements of car body as rigid beam, $\{y\}^T = \langle y_v \theta_v \rangle$, Chapter 7
YQ	wheelset lateral to vertical force ratio
y_v	vertical displacement of car body as rigid beam
z	a cross-sectional axis
β	coefficient related to variation of acceleration
$[\Gamma]$	constraint matrix
γ	coefficient related to numerical damping
γ_0	wavelength of corrugation
$\{\Delta d_u\}$	upper-part vehicle displacement increments
Δ_{st}	maximum static deflection of beam with hinge supports
Δt	time increment

Δy_1	displacement increment of wheel mass
$\{\Delta y\}$	displacement increments of sprung mass unit
$\{\Delta U_b\}$	displacement increments of bridge
$\{\Delta u_b\}$	displacement increments of bridge element
δ	delta function
θ_b	rotation about x axis of bridge element
θ_v	rotation of car body as rigid beam
$\theta_x, \theta_y, \theta_z$	rotations of a cross section of curved beam about three axes
κ	stiffness ratio of beam to elastic springs
$[\lambda]$	transformation matrix, Eq. (10.14)
λ_r	wavelength of track irregularity
λ_u	horizontal characteristic number of beam-Winkler foundation
λ_v	vertical characteristic number of beam-Winkler foundation system
μ_i	coefficient of friction for i th wheel
ξ	damping coefficient
ξ_n	damping coefficient of n th mode
ρ	density of beam
φ	subtended angle of curved beam, $\varphi = L/R$
φ_u	rotation of car body as rigid bar, Chapter 8
ϕ_n	n th vibration mode
$\{\phi\}_n$	n th vibration mode of structure
$\Psi(t)$	unified amplitude function as defined in Eq. (5.45)
$[\Psi_{uu}]$	equivalent matrix for upper part of car body, Eq. (8.10)
$[\Psi_{wu}]$	matrix as defined in Eq. (8.16a)
Ω	exciting frequency implied by the moving load
Ω	spatial frequency of track irregularity, Chapters 9 and 10
ω	frequency of vibration
ω_d	damped frequency
ω_{dn}	damped frequency of vibration of n th mode

ω_{h1}	fundamental frequency for horizontal plane of curved beam
ω_n	frequency of vibration of n th mode
ω_v	vertical vibration frequency of car body
ω_{v1}	fundament frequency of vertical vibration of curved beam
ω_0	frequency of vibration for beam with hinge supports
ω_θ	rotational vibration frequency of car body