Fields ***M***edallists' ***L***ectures 2nd Edition

World Scientific Series in 20th Century Mathematics

Published

- Vol. 1 The Neumann Compendium
edited by F. Bródy and T. Vámos
- Vol. 2 40 Years in Mathematical Physics
by L. D. Faddeev
- Vol. 3 Selected Papers of Yu I Manin
by Y. Manin
- Vol. 4 A Mathematician and His Mathematical Work
— Selected Papers of S. S. Chern
edited by S. Y. Cheng, P. Li and G. Tian
- Vol. 5 Fields Medallists' Lectures
edited by Sir Michael Atiyah and Daniel Iagolnitzer
- Vol. 6 Selected Logic Papers
by G. E. Sacks
- Vol. 7 Selected Papers of F. W. J. Olver (in 2 Volumes)
edited by Roderick Wong
- Vol. 8 The Collected Papers of Wei-Liang Chow
edited by S S Chern and V V Shokurov
- Vol. 9 Fields Medallists' Lectures, 2nd Edition
edited by Sir Michael Atiyah and Daniel Iagolnitzer

World Scientific Series in 20th Century Mathematics – Vol. 9

Fields Medallists' Lectures

2nd Edition

Editors

Sir Michael Atiyah

University of Edinburgh, UK

Daniel Iagolnitzer

Service de Physique Theorique

CEA-Saclay, France

World Scientific

New Jersey • London • Singapore • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: Suite 202, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

The editors and publisher would like to thank the following organisations and publishers of the various journals and books for their assistance and permission to reproduce the selected reprints found in this volume:

Academia Scientiarum Fennica

American Mathematical Society

Birkhäuser Verlag

Cambridge University Press

Canadian Mathematical Society

Documenta Mathematica

Dunod Editeur

Elsevier Science Publishers

Finnish Academy of Science and Letters

Higher Education Press

Institut Mittag-Leffler

International Mathematical Union

London Mathematical Society

Mathematical Society of Japan

Mir Publishers

Polish Scientific Publishers PWN Ltd.

Springer-Verlag

While every effort has been made to contact the publishers of reprinted papers prior to publication, we have not been successful in some cases. Where we could not contact the publishers, we have acknowledged the source of the material. Proper credit will be accorded to these publishers in future editions of this work after permission is granted.

Fields Medal on cover: photographed by AK Peters.

FIELDS MEDALLISTS' LECTURES, 2nd Edition

World Scientific Series in 20th Century Mathematics — Vol. 9

Copyright © 2003 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 981-238-256-9

ISBN 981-238-259-3 (pbk)

Printed in Singapore.

PREFACE TO THE FIRST EDITION

Although the Fields Medal does not have the same public recognition as the Nobel Prizes, they share a similar intellectual standing. It is restricted to one field — that of mathematics — and an age limit of 40 has become an accepted tradition. Mathematics has in the main been interpreted as pure mathematics, and this is not so unreasonable since major contributions in some applied areas can be (and have been) recognized with Nobel Prizes. The restriction to 40 years is of marginal significance, since most mathematicians have made their mark long before this age.

A list of Fields Medallists and their contributions provides a bird's eye view of mathematics over the past 60 years. It highlights the areas in which, at various times, greatest progress has been made. This volume does not pretend to be comprehensive, nor is it a historical document. On the other hand, it presents 22 Fields Medallists and so provides a highly interesting and varied picture.

The contributions themselves represent the choice of the individual Medallists. They are either reproductions of already published works, or are new articles produced for this volume. In some cases they relate directly to the work for which the Fields Medals were awarded. In other cases they relate to more current interests of the Medallists. This indicates that while Fields Medallists must be under 40 at the time of the award, their mathematical development goes well past this age. In fact the age limit of 40 was chosen so that young mathematicians would be encouraged in their future work.

The contribution of each medallist is in most cases preceded by the introductory speech given by another leading mathematician during the prize ceremony, a photograph and an up-to-date biographical notice. The introductory speech outlines the basic works of the medallist at the time of the medal and the reasons why it was awarded.

The Editors

This page is intentionally left blank

PREFACE TO THE SECOND EDITION

It is a pleasure to present this second edition which includes new contributions, by the 1998 and 2002 medallists.

Most Fields medallists are still living and most of them in full scientific activity. However, in addition to Lars Ahlfors (who was the first medallist with Jesse Douglas in 1936), we were very sad to learn the more recent death of other medallists. Due to the cooperation of his daughter, Mrs Oka, we are pleased to include a work by Kunihiro Kodaira, 1954 medallist who died in 1997 (the year of the first edition of this volume). On the other hand, Laurent Schwartz and Rene Thom also died recently, after the first edition of this book. Both were great scientists who made remarkable contributions to mathematics and other domains of science, and also deeply influenced younger colleagues through their teaching (one of us, D.I., was e.g. a student of Laurent Schwartz at the University of Paris). Both had a warm personality which attracted many friends.

We would like to dedicate this second edition to them.

The Editors

This page is intentionally left blank

RECIPIENTS OF FIELDS MEDALS

1936

Lars V. Ahlfors
Jesse Douglas

1950

Laurent Schwartz
Atle Selberg

1954

Kunihiko Kodaira
Jean-Pierre Serre

1958

Klaus F. Roth
René Thom

1962

Lars Hörmander
John W. Milnor

1966

Michael F. Atiyah
Paul J. Cohen
Alexander Grothendieck
Stephen Smale

1970

Alan Baker
Heisuke Hironaka
Sergei P. Novikov
John G. Thompson

1974

Enrico Bombieri
David B. Mumford

1978

Pierre R. Deligne
Charles L. Fefferman
Gregory A. Margulis
Daniel G. Quillen

1982

Alain Connes
William P. Thurston
Shing-Tung Yau

1986

Simon K. Donaldson
Gerd Faltings
Michael H. Freedman

1990

Vladimir G. Drinfeld
Vaughan F. R. Jones
Shigefumi Mori
Edward Witten

1994

Jean Bourgain
Pierre-Louis Lions
Jean-Christophe Yoccoz
Efim I. Zelmanov

1998

Richard E. Borcherds

William T. Gowers

Maxim Kontsevich

Curtis T. McMullen

2002

Laurent Lafforgue

Vladimir Voevodsky

CONTENTS

Preface to the First Edition	v
Preface to the Second Edition	vii
Recipients of Fields Medals	ix
1936 L. V. AHLFORS	
Biographical Notice	3
<i>Quasiconformal Mappings, Teichmüller Spaces, and Kleinian Groups</i>	5
1950 L. SCHWARTZ	
The Work of L. Schwartz by H. Bohr	19
Biographical Notice	25
<i>Calcul Infinitesimal Stochastique</i>	27
1954 K. KODAIRA	
Obituary: Kunihiko Kodaira	47
Biographical Notice	55
<i>On Kähler Varieties of Restricted Type</i>	57
1958 K. F. ROTH	
The Work of K. F. Roth by H. Davenport	61
Biographical Notice	67
<i>Rational Approximations to Algebraic Numbers</i>	68
R. THOM	
The Work of R. Thom by H. Hopf	75
Autobiography	79
1962 L. HÖRMANDER	
Hörmander's Work on Linear Differential Operators by L. Gårding	85

	Autobiography	91
	<i>Looking forward from ICM 1962</i>	94
1966	M. F. ATIYAH	
	L'oeuvre de Michael F. Atiyah by H. Cartan	113
	Biography	121
	<i>The Index of Elliptic Operators</i>	123
	S. SMALE	
	Sur Les Travaux de Stephen Smale by R. Thom	137
	Biographical Notice	143
	<i>A Survey of Some Recent Developments in Differential Topology</i>	150
1970	A. BAKER	
	The Work of Alan Baker by P. Turán	165
	Biography	169
	<i>Effective Methods in the Theory of Numbers</i>	170
	<i>Effective Methods in Diophantine Problems</i>	179
	<i>Effective Methods in Diophantine Problems. II</i>	191
	<i>Effective Methods in the Theory of Numbers/Diophantine Problems</i>	198
	S. NOVIKOV	
	The Work of Serge Novikov by M. F. Atiyah	203
	Biography	207
	<i>Rôle of Integrable Models in the Development of Mathematics</i>	210
1974	D. MUMFORD	
	The Work of David Mumford by J. Tate	227
	Autobiography	233
	<i>Pattern Theory: A Unifying Perspective</i>	234
1978	G. A. MARGULIS	
	The Work of Gregory Aleksandrovitch Margulis by J. Tits	271
	Biographical Notice	279
	<i>Oppenheim Conjecture</i>	281

1982	A. CONNES	
	The Work of Alain Connes by H. Araki	337
	Biographical Notice	347
	<i>Brisure de Symétrie Spontanée et Géométrie du Point de vue Spectral</i>	348
	W. P. THURSTON	
	The Work of W. Thurston by C. T. C. Wall	381
1986	S. K. DONALDSON	
	The Work of Simon Donaldson by M. F. Atiyah	385
	Biographical Notice	391
	<i>Remarks on Gauge Theory, Complex Geometry and 4-Manifold Topology</i>	392
	M. H. FREEDMAN	
	The Work of M. H. Freedman by J. Milnor	413
	Biographical Notice	419
	<i>Betti Number Estimates for Nilpotent Groups</i>	421
1990	V. F. R. JONES	
	The Work of Vaughan F. R. Jones by J. S. Birman	443
	Biographical Notice	455
	<i>A Polynomial Invariant for Knots via von Neumann Algebras</i>	456
	<i>Index for Subfactors</i>	467
	S. MORI	
	The Work of Shigefumi Mori by H. Hironaka	495
	Biographical Notice	503
	<i>Birational Classification of Algebraic Threefolds</i>	504
	E. WITTEN	
	The Work of E. Witten by L. D. Faddeev	519
	The Work of Edward Witten by M. F. Atiyah	522
	Biographical Notice	529
	<i>Geometry and Quantum Field Theory</i>	531

1994	J. BOURGAIN	
	The Work of Jean Bourgain by L. Caffarelli	545
	Biographical Notice	549
	<i>Hamiltonian Methods in Nonlinear Evolution Equations</i>	550
	P. L. LIONS	
	The Work of Pierre-Louis Lions by S. R. S. Varadhan	563
	Biographical Notice	568
	<i>On Some Recent Methods for Nonlinear Partial Differential Equations</i>	572
	J. C. YOCCOZ	
	Présentation de Jean-Christophe Yoccoz by A. Douady	589
	<i>Recent Developments in Dynamics</i>	596
	E. I. ZELMANOV	
	The Work of Efim Zelmanov by W. Feit	619
	Biographical Notice	629
	<i>On the Restricted Burnside Problem</i>	631
1998	R. E. BORCHERDS	
	The Work of Richard Ewen Borchers by P. Goddard	641
	Biographical Notice	653
	<i>What is Moonshine?</i>	654
	W. T. GOWERS	
	The Work of William Timothy Gowers by B. Bollobás	663
	Autobiography	673
	<i>A New Proof of Szemerédi's Theorem for Arithmetic Progressions of Length Four</i>	678
	M. KONTSEVICH	
	The Work of Maxim Kontsevich by C. H. Taubes	703
	Biographical Notice	713

<i>Formal (Non)-Commutative Symplectic Geometry</i>	716
<i>Comments on “Formal (Non)-Commutative Symplectic Geometry”</i>	734

C. T. McMULLEN

The Work of Curtis T. McMullen by J. Milnor	737
Biographical Notice	745
<i>Rigidity and Inflexibility in Conformal Dynamics</i>	746

2002 L. LAFFORGUE

The Work of Laurent Lafforgue by G. Laumon	761
--	-----

V. VOEVODSKY

The Work of Vladimir Voevodsky by C. Soulé	769
Biographical Notice	775
<i>Open Problems in the Motivic Style Homotopy Theory, I</i>	779