Vertical Density Representation and Its Applications

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Dedications

To the memory of my Mother, to my Father, to Mark, Marleen and Andrew, and to Helen.

To my parents, to my wife, Sui-Ching, to Wai-Ying and Chun-Yan.

To my parents and my family.

MDT, WKP, SHH

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Preface

Since the first paper related to Vertical Density Representation (VDR) appeared in 1991 several papers have been published and work continues on the topic. VDR arose in connection with the analysis of performance measurement. Consideration of the Box-Muller method for generation of normal variates led to a question about the density (probability density function, or pdf) of the density function itself. The purpose of this book is to survey these results and provide some new unpublished results. We hope to have made contact with the major published articles on the topic. While it is possible that some could have been missed, we have made a diligent search and have sought the advice of other researchers on the topic.

VDR may be regarded as a special kind of variable transformation but may also be considered as a more general density-modeling tool. By assuming that a variate is uniformly distributed on the contours or level curves of a given function in real n -dimensional space, and considering the density of the ordinate of the given function, the density of the original variate can be represented. Basic results and extensions are discussed. Several applications are described for use with Monte Carlo simulation. The uniformly distributed on contours assumption can be relaxed with what we call the General VDR Theorem. An application of that result is made to a problem in inverse linear programming.

A new result on the analysis of correlation into two distinct components is given along with its potential value in the aggregation of experts. Also a new result on densities of orbit values produced by chaos generators is given along with the construction of a large class of chaos-based uniform random number generators. We also provide a case study in applying VDR

in connection with what may be called behavioral estimation. VDR considerations are brought to bear in developing a validation technique for the estimation method. Then a further application to estimating benchmark costs and cost matrices is given. Finally in the last chapter, we discuss some future research questions and work in progress for several of the topics discussed in the earlier chapters.

In order to have a good understanding of the book, a background in mathematics and statistics at the graduate level will be best. However, readers having had a calculus-based statistics course and some acquaintance with Lebesgue measure should fare reasonably well. In the past, statistical theory has depended primarily on the Riemann integral. VDR depends more heavily on Lebesgue measure. However, the derivations are kept intuitive with respect to those issues in so far as possible.

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