QUANTITATIVE FINANCE AND RISK MANAGEMENT

A Physicist's Approach

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A Physicist's Approach

Jan W Dash

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Dedication

I dedicate this book to my father and mother, Edward and Honore Dash. They inspired learning and curiosity, and advised never to take anything for granted.

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