Algorithms

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Design Techniques and Analysis

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To my wife Noura

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Preface

The field of computer algorithms has flourished since the early 1960's when the first users of electronic computers started to pay attention to the performance of programs. The limited resources of computers at that time resulted in additional impetus for devising efficient computer algorithms. After extensive research in this field, numerous efficient algorithms for different problems emerged. The similarities among different algorithms for certain classes of problems have resulted in general algorithm design techniques. This book emphasizes most of these algorithm design techniques that have proved their utility in the solution to many problems. It may be considered as an attempt to cover the most common techniques in the design of sequential algorithms. Each technique is presented as follows. First, the context in which that technique can be applied. Second, the special characteristics of that technique that set it apart. Third, comparison with other techniques, whenever possible; finally, and most importantly, illustration of the technique by applying it to several problems.

Although the main theme of the book is algorithm design techniques, it also emphasizes the other major component in algorithmic design: the analysis of algorithms. It covers in detail the analysis of most of the algorithms presented. Chapter 2 covers most of the mathematical tools that are helpful in analyzing algorithms. Chapter 11 is an introduction to the field of computational complexity, and Chapter 12 covers the basics of establishing lower bounds on the solution of various problems. These chapters are indispensable for the design of efficient algorithms.

The focus of the presentation is on practical applications of the design techniques. Each technique is illustrated by providing an adequate num-

ber of algorithms to solve some problems that quite often arise in many applications in science and engineering.

The style of presentation of algorithms is straightforward, and uses pseudocode that is similar to the syntax of structured programming languages, e.g. **if-then-else**, **for** and **while** constructs. The pseudocode is sometimes intermixed with English whenever necessary. Describing a portion of an algorithm in English is indeed instructive; it conveys the idea with minimum effort on the part of the reader. However, sometimes it is both easier and more formal to use a pseudocode statement. For example, the function of the assignment statement

$$B[1..n] \leftarrow A[1..n]$$

is to replace each entry $B[i]$ with $A[i]$ for all $i, 1 \leq i \leq n$. Neither the **for ... end for** construct nor plain English is more concise or easier to state than this notation.

The book is divided into seven parts. Each part consists of chapters that cover those design techniques that have common characteristics or objectives. Part 1 sets the stage for the rest of the book, in addition to providing the background material that is needed in subsequent chapters. Part 2 is devoted to the study of recursive design techniques, which are extremely important, as they emphasize a fundamental tool in the field of computer science: recursion. Part 3 covers two intuitive and natural design techniques: the greedy approach and graph traversals. Part 4 is concerned with those techniques needed to investigate a given problem and the possibility of either coming up with an efficient algorithm for that problem, or proving its intractability. This part covers NP-completeness, computational complexity and lower bounds. In Part 5, techniques for coping with hard problems are presented. These include backtracking, randomization and finding approximate solutions that are reasonable and acceptable using a reasonable amount of time. Part 6 introduces the concept of iterative improvement using two important problems that have received extensive attention, which resulted in increasingly efficient algorithms: the problem of finding a maximum flow in a network and the problem of finding a maximum matching in an undirected graph. Finally, Part 7 is an introduction to the relatively new field of computational geometry. In one chapter, the widely used technique of geometric sweeping is presented with examples of important problems in that field. In the other chapter, the versatile tool of

the Voronoi diagram is covered, and some of its applications are presented.

The book is intended as a text in the field of the design and analysis of algorithms. It includes adequate material for two courses in algorithms. Chapters 1 through 10 provide the core material for an undergraduate course in algorithms at the junior or senior level. Some of the material may be skipped such as the amortized analysis of the union-find algorithms, and the linear time algorithms in the case of dense graphs for the shortest path and minimum spanning tree problems. The instructor may find it useful to add some of the material in the following chapters such as backtracking, randomized algorithms, approximation algorithms or geometric sweeping. The rest of the material is intended for a graduate course in algorithms.

The prerequisites for this book have been kept to the minimum; only an elementary background in discrete mathematics and data structures are assumed.

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Dhahran, Saudi Arabia

M. H. Alsuwaiyel

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