QUANTUM HALL EFFECTS

Field Theoretical Approach and Related Topics

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FIELD THEORETICAL APPROACH AND RELATED TOPICS

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To Shizu & Motohiko

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Preface

Quantum Hall (QH) effects are remarkable macroscopic quantum phenomena observed in the 2-dimensional electron system. The integer QH effect was discovered in 1980 by K. von Klitzing, a century after the discovery of the classical Hall effect, for which he received the Nobel prize in 1985. The fractional QH effect was discovered in 1982 by D. Tsui, H. Störmer and A.C. Gossard. It was predicted by B. Laughlin that a quasiparticle is an anyon carrying electric charge e/m at the filling factor $\nu = 1/m$. In 1997 a direct observation of fractional charges was successfully carried out at $\nu = 1/3$ by measuring a back scattering current noise in Hall-bar experiments. B. Laughlin, D. Tsui and H. Störmer received the Nobel prize in 1998.

QH effects are so special in condensed matter physics that they are deeply connected with fundamental principles of physics. Moreover, they present concrete realizations of various modern concepts related with topological investigations not only in physics but also in mathematics. The QH system provides us with a rare opportunity to enjoy the interplay between condensed matter physics and particle physics. It is worthwhile to make the subject as a part of the training for all graduate students in physics.

Many fancy ideas appear in QH effects. Composite particle (boson or fermion) is one of them. It is an electron bound to flux quanta. Laughlin started his seminal paper by saying that "The $\frac{1}{3}$ effect, recently discovered by Tsui, Störmer and Gossard, results from the condensation of the two-dimensional electron gas in a GaAs-Ga_xAl_{1-x} heterostructure into a new type of collective ground state". It is our present understanding that the QH state is a condensate of composite bosons. Intriguingly a single electron is converted into a boson by acquiring flux quanta in QH states. Such a statistical transmutation is allowed in the planar geometry due to its intrinsic topological structure. Despite their bosonic low-energy properties, composite bosons obey the fermionlike exclusion principle. The hierarchy of fractional QH states is understood by the use of composite fermions.

Topological solitons play the leading role in QH effects. Indeed, charged excitations (quasiparticles) are topological solitons in the QH condensate. When quantum coherence develops with the spin wave as a Goldstone mode, quasiparticles are skyrmion O(3)-spin textures. Skyrmions were originally proposed in nuclear physics, where they are O(4)-isospin textures to be identified with nucleons. Though their relevance is still unclear in nuclear physics, their existence is firmly established in the QH system.

Edge excitations are described by the chiral Tomonaga-Luttinger model. Electrons are topological excitations in this model, and obey a relativistic field equation. When tunneling interactions are allowed between the opposite edges, topological excitations turn into sine-Gordon solitons. Physics on the edge is "exactly solvable", due to the existence of infinitely many conservation rules. It can be a laboratory to test various results of conformal field theories.

The bilayer QH system consists of two quantum wells separated by a barrier with ~ 30 nm width. A phase transition has been observed between two distinguishable phases at a fixed filling factor by changing the electron density. When the tunneling gap is not too large, interlayer coherence develops, as is a reminiscence of the superconductor Josephson junction. Pseudoparticles are CP^3 skyrmions.

Physics confined to the lowest Landau level is curious, because the x and y coordinates of the electron position become noncommutative. It is a simplest physical system subject to noncommutative geometry. The system is characterized by the Moyal algebra or the W_∞ algebra. When restricted to the edge of the QH droplet, the Kac-Moody algebra emerges. The excitation spectrum of the QH system forms a representation of the algebra.

This book is intended to give a pedagogical and self-contained introduction to these new concepts in QH effects. It is accessible, and will be of interest, to students and researchers in condensed matter physics, particle and mathematical physics. It comprises four parts. Part I is a quick summary of quantum field theory, where I explain various concepts necessary to understand QH effects, namely, canonical quantization, quantum coherence, topological solitons, anyons and so on. I hope this part is useful to those who are not familiar to these concepts. Readers may skip this part and come back to relevant places when necessary. Part II is devoted to monolayer QH systems, while Part III to bilayer QH systems. Some algebraic aspects of the QH system are reviewed in Part IV, where I derive some basic formulas used in Part II and Part III.

I have benefited from fruitful collaborations on the subject with A. Sawada, A. Iwazaki, H. Ohno, Y. Hirayama, K. Muraki, Y. Horikoshi, Y. Ohno, T. Saku, N. Kumada, M. Hotta, K. Sasaki, K. Hasebe, Y-S. Wu and others. I am grateful to H. Aoki, D. Yoshioka, A. MacDonald, S. Murphy and J. Eisenstein for valuable discussions. Special thanks are due to K. Shizuya, S. Katsumoto and K. Takahashi for careful reading of the manuscript. Their comments were very helpful for me to improve the manuscript.

Zyun Francis Ezawa
Sendai
January 2000

Quantum Hall Effect

Fig. 1 Hall and diagonal resistivities R_{xy} and R_{xx} are independent of sample properties and given by $R_{xy} = V_y/J_{\text{tot}} \rightarrow \nu^{-1}(2\pi\hbar/e^2)$, $R_{xx} = -WV_x/LJ_{\text{tot}} \rightarrow 0$, in quantum Hall (QH) states. Here, ν is the filling factor.

Fig. 2 QH states are detected by plateaux developed in the Hall resistivity R_{xy} or dips in the diagonal resistivity R_{xx} .

Fig. 3 There are m flux quanta (vertical lines) per electron at $\nu = 1/m$. A composite particle (fermion or boson) is an electron bound to $(m-1$ or $m)$ flux quanta. It behaves as a "fat" electron in the fractional QH state. Here, (a) $m = 1$ and (b) $m = 3$.

Skyrmions

Fig. 4 (a) An up-spin electron is excited to the down-spin level thermally at $\nu = 1$, leaving a quasihole behind. The excitation energy consists of the exchange Coulomb energy, the direct Coulomb energy and the Zeeman energy. (b) The direct Coulomb energy is reduced by exciting neighboring electrons coherently. (c) The excited objects are a skyrmion with charge $+e$ and an antiskyrmion with charge $-e$. They are topological solitons in QH ferromagnets. The size is determined so as to optimize the Coulomb and Zeeman energies. (c) The electron densities $\rho^\downarrow(x)$ and $\rho^\uparrow(x)$ are illustrated around them. (d) The electron spin is reversed at the center of the skyrmion and the antiskyrmion. The normalized spin vector is depicted.

Edge States

Fig. 5 Dispersion relation is illustrated in a QH bar as a function of the one-dimensional momentum $\hbar k$, related to the electron position as $y = \ell_B^2 k$ in the Landau gauge. Gapless edge channels appear when the Fermi level crosses energy levels.

Fig. 6 Quasiparticles flow along the edge. With weak pinch-off by bias voltages, they tunnel between the top and bottom edges through the QH liquid.

Fig. 7 The charge of quasiparticles can be determined by the Shchottky noise formula, when they tunnel one by one as in Fig.6. Tunneling noise was measured in the QH state at the bulk filling factor $\nu = 2/3$, and also at $\nu = 4$ (given in the inset). The charge $e/3$ of quasiparticles was clearly observed at $\nu = 1/3$. Data are taken from the work due to Saminadayar *et al.*^[340]

Bilayer QH System

Fig. 8 (a) The bilayer system has four scale parameters, the magnetic length ℓ_B , the Zeeman gap Δ_Z , the interlayer distance d and the tunneling gap Δ_{SAS} . A parallel magnetic field $B_{\parallel}$ may be additionally applied to the system by tilting the sample. (b) A typical sample structure is illustrated. Electrons are provided by two remote δ -doped donor layers separated by AlGaAs spacers. (c) The number density ρ^{α} in each quantum well is controlled by applying gate bias voltages.

Fig. 9 Hall resistivity is plotted at a fixed total density ρ : Each curve is obtained (a) for a fixed value of the imbalance parameter σ , and (b) for a fixed value of the tilted angle Θ . The imbalance parameter is defined by $\sigma = (\rho^f - \rho^b)/\rho$ with ρ^f (ρ^b) the density in the front (back) layer. The data are taken from Sawada *et al.* [343,345]

Pseudospins

Fig. 10 The pseudospin $\mathcal{J} = (\mathcal{J}_x, \mathcal{J}_y, \mathcal{J}_z)$ is defined by assigning $\mathcal{J}_z = \pm 1/2$ to electrons in the front or back layer. The pseudomagnetic field $\mathcal{B} = (\Delta_{\text{SAS}}, 0, eV_{\text{bias}})$ is "tilted" by applying the bias voltage V_{bias} , as rotates the pseudospin $\mathcal{J}$ so that the pseudo-Zeeman energy $H_{\text{PZ}} \propto -\mathcal{B}\mathcal{J}$ is minimized. This is the process of a charge transfer between the two layers. When a bilayer QH state is stable (unstable) against this charge transfer, it belongs to a continuous (discrete) spectrum of $\mathcal{J}_z$.

Fig. 11 Various bilayer QH states are illustrated in terms of composite particles. In these three figures the magnetic field is the same but the electron densities are different. (a) Compound state at $\nu = 2/3$: The QH state develops independently in each layer. (b) Bilayer-locked state at $\nu = 1/2$: The Coulomb repulsive force repels electrons on the opposite layer and creates a "hole" as indicated by an open oval. (c) Charge-transferable state at $\nu = 1/3$: Electrons do not have home layers since they tunnel freely between the two layers. Due to the interlayer exchange interaction the interlayer coherence may develop spontaneously on this state.

Compound and Charge-Transferable States (Theory)

Fig. 12 A bilayer QH state belongs either to a discrete spectrum or a continuous spectrum of the pseudospin component J_z . Various spectra appear (a) for $\Delta_{SAS} = 0$, (b) for $0 \neq \Delta_{SAS} < \Delta_Z$ and (c) for $\Delta_{SAS} > \Delta_Z$. A solid circle on lattice point (ν^b, ν^f) denotes a *compound state* at $\nu = \nu^b + \nu^f$, where the tunneling gap collapses. (a') It is made of the $\nu = \nu^b$ and $\nu = \nu^f$ monolayer QH states. (b&c) A heavy line indicates a charge-transferable (*coherent* or *resonant*) state. Charge transfer is forbidden in the pseudospin singlet state. (b'&c') By the tunneling gap one Landau level may be split into four energy levels, the up-spin symmetric ($S\uparrow$) and antisymmetric ($A\uparrow$) states, the down-spin symmetric ($S\downarrow$) and antisymmetric ($A\downarrow$) states. A heavy line represents a filled level. The symbol $\odot$ indicates a charge-transferable level.

Compound and Charge-Transferable States (Experiments)

Fig. 13 Diagonal resistance was measured by scanning both the front and back gate voltages in a bilayer system. The bright regions correspond to small resistance, indicating QH states [See Fig.12]. (a) In a sample with $\Delta_{SAS} \approx 1$ K, only compound states are seen for $\nu \geq 2$. (b) In the same sample, the $\nu = 1$ state is elongated, indicating a coherent state. (c) & (d) In a sample with $\Delta_{SAS} \approx 11$ K, a characteristic pattern $\{1,2,1,0\}$ of charge transferable states is prominent besides compound states. The data are taken from Muraki *et al.*.^[288]

Bilayer QH States at $\nu = 1$

Fig. 14 (a) A phase diagram is illustrated at $\nu = 1$, where QH state exists only when the total density ρ is smaller than a certain value ($\rho > \rho_{QH}$) in a given sample. Interlayer coherence will develop provided Δ_{SAS}/E_C^0 is small. The horizontal segment at $\rho = 0.6$ implies that the sample was tilted at this density. (b) As ρ increases, activation energy decreases and collapses at $\rho_{QH} = 1.5 \times 10^{11} \text{ cm}^{-2}$. Here, sample parameters are $d = 23 \text{ nm}$ and $\Delta_{SAS} = 6.7 \text{ K}$. (c) As σ increases, activation energy increases due to the increase of the capacitance energy. (d) As the sample is tilted, activation energy decreases anomalously due to an interlayer-coherence effect. The data are taken from Sawada *et al.* [344,345]

Bilayer QH States at $\nu = 2$

Fig. 15 (a) A phase diagram is illustrated at $\nu = 2$, where there are two distinctive QH states [Fig.12]. It is a compound state (ferromagnet) for $\rho > \rho_{\text{QH}}$, and a charge-transferable state (spin singlet) for $\rho < \rho_{\text{QH}}$. Interlayer coherence develops provided $\Delta_{\text{SAS}}/E_C^0$ is small. (b) Activation energy as a function of ρ shows that $\rho_{\text{QH}} = 0.9 \times 10^{11} \text{ cm}^{-2}$ in this sample ($d = 23 \text{ nm}$, $\Delta_{\text{SAS}} = 6.7 \text{ K}$). (c) As σ increases, activation energy increases smoothly in the charge-transferable state, but decreases rapidly in the compound state; the compound state is stable only at the balanced point. (d) As the sample is tilted, activation energy decreases in the charge-transferable state due to an interlayer-coherence effect, but increases in the compound state due to the Zeeman effect. The data are taken from Sawada *et al.* [344,345]

Skyrmions in Bilayer QH States

Fig. 16 A skyrmion excitation in the bilayer system modulates electron densities in both layers. The capacitance energy is proportional to σ^2 , where σ is the imbalance parameter.

Fig. 17 Activation energy is given as a function of σ . At $\nu = 1$ the data are explained by the capacitance energy of a skyrmion with charge e , while at $\nu = 2$ by the capacitance and tunneling energies of a skyrmion with charge $2e$. The data are the same ones in Figs.14(c) and 15(c) due to Sawada *et al.*^[344]

Contents

<i>Preface</i>	vii
----------------------	-----

Part. I Quantum Field Theory	1
-------------------------------------	----------

1 Quantum Mechanics	3
1.1 Hilbert Spaces	3
1.2 Hamiltonian Formalism	4
1.3 Creation and Annihilation Operators	6
1.4 Uncertainty Principle	10
1.5 Coherent States and Squeezed States	11
1.6 Particle Number and Phase	14
1.7 Macroscopic Coherence	16
2 Quantum Field Theory	18
2.1 One-Body Hamiltonian	18
2.2 Many-Body Hamiltonian	19
2.3 Boson Field Operators	21
2.4 Quantum Field Theory	22
2.5 Fermion Field Operators	24
3 Canonical Quantization	26
3.1 Relativistic Particles and Waves	26
3.2 Schrödinger Field	27
3.3 Real Klein-Gordon Field	32
3.4 Complex Klein-Gordon Field	37
3.5 Nöther Currents	39
4 Spontaneous Symmetry Breaking	41
4.1 Ferromagnets	41
4.2 Real Klein-Gordon Field	45
4.3 Complex Klein-Gordon Field	49
4.4 Sigma Model	51

4.5	Schrödinger Field	53
4.6	Superfluidity	57
4.7	Goldstone Theorem	59
5	Electromagnetic Field	63
5.1	Maxwell Equations	63
5.2	Canonical Quantization	66
5.3	Interaction with Matter Field	71
5.4	Anderson-Higgs Mechanism	73
5.5	Massive Vector Field	75
5.6	Superconductivity	77
6	Topological Solitons	83
6.1	Topological Sectors	83
6.2	Classical Fields	85
6.3	Solitary Waves, Kinks and Solitons	88
6.4	Sine-Gordon Solitons	90
6.5	Vortex Solitons	94
6.6	Homotopy Classes	100
6.7	Skyrmions in Planar Geometry	105
6.8	CP^{N-1} Skyrmions	110
7	Anyons	116
7.1	Spin and Statistics	116
7.2	Fractional Statistics	121
7.3	Quantum Mechanics	122
7.4	Aharonov-Bohm Effect	126
7.5	Chern-Simons Gauge Theory	130
7.6	Anyon Field Operators	134

Part. II	Monolayer Quantum Hall Systems	137
-----------------	---------------------------------------	------------

8	Introduction	139
9	Landau Quantization	153
9.1	Planar Electrons	153
9.2	Cyclotron Motion	157
9.3	Hall Current	162
9.4	Symmetric Gauge	163
9.5	Landau Gauge	168
9.6	Lowest-Landau-Level Condition	173

10 Quantum Hall Effects	176
10.1 Incompressibility	176
10.2 Integer Quantum Hall Effects	177
10.3 Fractional Quantum Hall Effects	179
10.4 Quasiparticles.....	181
10.5 Hall Plateaux	185
11 Quasiparticles and Activation Energy	187
11.1 Impurity Potentials	187
11.2 Gap Energies.....	188
11.3 Dispersion Relation	192
11.4 Thermal Activation	195
12 Field Theory of Composite Particles	200
12.1 Composite Particles	200
12.2 Statistical Transmutation	203
12.3 Effective Magnetic Field	206
12.4 Dressed Composite Particles	209
12.5 Composite Particles in Lowest Landau Level	213
12.6 Composite Fermions in Lowest Landau Level	216
13 Composite Bosons and Semiclassical Analysis	221
13.1 Semiclassical Ground State	221
13.2 Vortex Excitations	223
13.3 Field Theory of Vortex Solitons	231
13.4 Haldane-Halperin Hierarchy	234
14 Quantum Hall Ferromagnets	238
14.1 Spin Coherence	238
14.2 Spin Degree of Freedom.....	240
14.3 SU(2) Composite Bosons	242
14.4 Wave Functions	243
14.5 Spin Field, Sigma Field and CP ¹ Field	244
14.6 Effective Hamiltonian	249
15 Spin Textures	252
15.1 Spin Excitations	252
15.2 Skyrmions	256
15.3 Skyrmion Energy	262
16 Hierarchy of Fractional Quantum Hall States	268
16.1 Jain Hierarchy	268
16.2 Landau Levels of Composite Fermions	271
16.3 Beyond Principal Sequences	273

16.4 Spin Polarization	276
16.5 Gap Energies	280
17 Edge Effects	285
17.1 Shot Noises of Fractional Charges	285
17.2 Chiral Edge Excitations	288
17.3 Chiral Luttinger Liquid	290
17.4 Electrodynamics on Edge	295
17.5 Edge Tunneling and Sine-Gordon Solitons	299

Part. III Bilayer Quantum Hall Systems	305
---	------------

18 Introduction	307
19 SU(2) Pseudospin Structure	320
19.1 Bilayer Planar Electrons	320
19.2 Gate Bias Voltages	323
19.3 Pseudospins	325
19.4 Compound States	329
19.5 Charge-Transferable States	333
19.6 Phase Diagrams	337
20 Bilayer-Locked States	340
20.1 Composite-Boson Field	340
20.2 Wave Functions	344
20.3 Ground State	345
20.4 Vortex Excitations	346
21 Interlayer-Coherent States	349
21.1 Interlayer Coherence	349
21.2 Composite Bosons	350
21.3 CP^1 Field	353
21.4 Ground State	354
21.5 Goldstone Mode	356
21.6 Pseudospin Textures	359
22 Skyrmions in SU(4)-Invariant Regime	364
22.1 SU(4)-Invariant Regime	364
22.2 CP^3 Skyrmions	368
22.3 Spin-Skyrmions at $\nu = 1$	371
22.4 Spin-Pseudospin Coherence at $\nu = 2$	375
22.5 Compound State at $\nu = 2$	379

23 Bilayer Quantum-Hall Junction	381
23.1 Josephson-Like Phenomena	381
23.2 Parallel Magnetic Field	382
23.3 Effective Hamiltonian	385
23.4 Commensurate-Incommensurate Phase Transition	390
23.5 Josephson Junction versus Quantum-Hall Junction	396
23.6 Plasmon Excitations	399
23.7 Josephson-Like Effects	401

Part. IV Algebraic Approach	405
------------------------------------	------------

24 Introduction	407
25 Lowest-Landau-Level Projection	412
25.1 Quantum Mechanics	412
25.2 Quantum Field Theory	415
25.3 Magnetic Translation	418
25.4 Moyal Algebra	421
25.5 Edge States	424
26 Quantum Hall Effects in Algebraic Approach	427
26.1 Spinless Quantum Hall States	427
26.2 Hall Current	429
26.3 Nöther Currents	432
26.4 $W_\infty \times SU(2)$ Density Algebra	436
26.5 Monolayer Quantum Hall Ferromagnets	437
26.6 Spin Textures	439
27 Effective Hamiltonian in Algebraic Approach	442
27.1 Bilayer Quantum Hall States	442
27.2 Static Correlation Functions	444
27.3 Effective Hamiltonian for Coherent Modes	446
28 Electric Currents in Bilayer Systems	452
28.1 Electric Currents	452
28.2 Hall Current	453
28.3 Screening Current	455
28.4 Tunneling Current	457

Appendices	459
A Hausdorff Formulas	459
B Pauli Matrices	459
C Cauchy-Riemann Equations	460
D Bogoliubov Transformation	461
E Energy-Momentum Tensor	464
F Exchange Interaction	465
G One-Dimensional Soliton Solutions	471
H Field-Theoretical Vortex Operators	473
I Perturbative Analysis	476
J Bosonization in One-Dimensional Space	479
References	485
Index	499