**HANDBOOK of
GRAPH GRAMMARS and
COMPUTING by GRAPH
TRANSFORMATION**

HANDBOOK OF GRAPH GRAMMARS AND COMPUTING BY GRAPH TRANSFORMATION

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Volume 1
FOUNDATIONS

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Preface

Graph grammars originated in the late 60's, motivated by considerations about pattern recognition, compiler construction, and data type specification. Since then the list of areas which have interacted with the development of graph grammars has grown impressively. Besides the aforementioned areas it includes software specification and development, VLSI layout schemes, database design, modelling of concurrent systems, massively parallel computer architectures, logic programming, computer animation, developmental biology, music composition, visual languages, and many others. Graph grammars are interesting from the theoretical point of view because they are a natural generalization of formal language theory based on strings and the theory of term rewriting based on trees.

The wide applicability of graph grammars is due to the fact that graphs are a natural way of describing complex situations on an intuitive level. Moreover, the graph transformations associated with a graph grammar bring “dynamic behaviour” to such descriptions, because they model the evolution of graphical structures. Therefore graph grammars and graph transformations become attractive as a “programming paradigm” for software and graphical interfaces.

Over the last 25-odd years graph grammars have developed at a steady pace into a theoretically sound and well-motivated research field. In particular, they are now based on solid foundations, presented in this volume. It includes a state-of-the-art presentation of the foundations of all basic approaches to graph grammars and computing by graph transformations.

The two most basic choices for rewriting a graph are *node replacement* and *edge replacement*, or, in the more general setting of hypergraphs, *hyperedge replacement*. In a node replacement graph grammar a node of a given graph is replaced by a new subgraph which is connected to the remainder of the graph by new edges depending on how the node was connected to it. In a hyperedge replacement graph grammar a hyperedge of a given hypergraph is replaced by a new subhypergraph which is glued to the remainder of the hypergraph by fusing (identifying) some nodes of the subhypergraph with some nodes of the remainder of the hypergraph depending on how the hyperedge was glued to it. Chapter 1 surveys the theory of node replacement graph grammars concentrating mainly on “context-free” (or “confluent”) node replacement graph grammars, while Chapter 2 surveys the theory of hyperedge replacement graph grammars. Both types of graph grammars naturally generalize the context-free string grammars.

The gluing of graphs plays also a central role in the *algebraic approach* to graph transformations, where a subgraph, rather than a node or an edge only, can be replaced by a new subgraph (generalizing in this way arbitrary type-0 Chomsky string grammars). Originally, the rewriting of graphs based on gluing has been formulated by the so-called *double pushout* in the category of graphs and total graph morphisms. More recently a *single pushout* in the category of graphs and partial graph morphisms has been used for this purpose. Chapter 3 gives an overview of the double pushout approach, and Chapter 4 gives an overview of the single pushout approach; it also presents a detailed comparison of the two approaches.

Graphs may be considered as *logical structures* and so one can express formally their properties by logical formulas. Consequently classes of graphs may be described by formulas of appropriate logical languages. Such logical formulas are used as finite devices, comparable to grammars and automata, to specify classes of graphs and to deduce properties of such classes from their logical descriptions. Chapter 5 surveys the relationships between monadic second-order logic and the context-free graph grammars of Chapters 1 and 2.

The research on graph transformations leads to a careful re-thinking of the formal framework that should be used for the specification of graphs. The theory of *2-structures*, a specific relational framework, has turned out to be fruitful for the investigation of graphs especially in their relationship to graph transformations. The “static part” of the theory of 2-structures allows to obtain rather strong decomposition results for graphs, while the “dynamic part” of the theory employs group theory to consider transformations of graphs as encountered in networks. Chapter 6 presents the basic theory of 2-structures.

In order to specify classes of graphs and graph transformations as they occur in various applications (e.g. databases and database manipulations) one often needs quite powerful extensions of the basic mechanisms of graph replacement systems. One such extension is to control the order of application of the graph replacement rules. Chapter 7 presents a basic framework for *programmed graph replacement* systems based on such a control.

We believe that this volume together with the two forthcoming volumes - on specification and programming, and on concurrency - provide the reader with a rather complete insight into the mathematically challenging area of graph grammars which is well motivated by its many applications to computer science and beyond.

My thanks go first of all to the graph grammar community - the enthusiastic group of researchers, often with very different backgrounds, spread all over the world - for providing a very stimulating environment for the work on graph grammars. Perhaps the main driving force in this community during the last 7 years was the ESPRIT Basic Research Working Group COMPUGRAPH (Computing by Graph Transformation) in the period March 1989 - February 1992, followed by the ESPRIT Basic Research Working Group COMPUGRAPH II in the period October 1992 - March 1996. The initial planning of the handbook took place within the COMPUGRAPH project, and most of Volume I of the handbook has been written within the COMPUGRAPH period. The European Community is also founding now the TMR network GETGRATS (General Theory of Graph Transformation Systems) for the period from September 1996 to August 1999. We hope to complete volumes II and III of the handbook within the GETGRATS project. The gratitude of the graph grammar community goes to the European Community for the generous support of our research. We are also indebted to H. Ehrig and his Berlin group for managing the COMPUGRAPH and COMPUGRAPH II Working Groups, and to A. Corradini and U. Montanari for their work in preparing the GETGRATS project.

I am very grateful to all the authors of the chapters of this volume for their cooperation, and to the Advisory Board: B. Courcelle, H. Ehrig, G. Engels, D. Janssens, H.-J. Kreowski and U. Montanari, for their valuable advice.

Finally, very special thanks go to Perdita Lhr for her help in transforming all the separate chapters into a homogeneous and readable volume as it is now. Neither she nor myself could foresee how much work it would be - but due to her efforts we could bring the project to a happy end.

G. Rozenberg
Managing Editor
Leiden, 1996

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Contents

1	Node Replacement Graph Grammars	1
	(J. Engelfriet, G. Rozenberg)	1
1.1	Introduction	3
1.2	From NLC to edNCE	4
1.2.1	Node replacement and the NLC methodology	4
1.2.2	Extensions and variations: the edNCE grammar	9
1.2.3	Graph replacement grammars	14
1.2.4	Bibliographical comments	15
1.3	Node replacement grammars with Neighbourhood Controlled Embedding	16
1.3.1	Formal definition of edNCE graph grammars	16
1.3.2	Leftmost derivations and derivation trees	38
	Leftmost derivations	38
	Derivation trees	43
1.3.3	Subclasses	55
1.3.4	Normal forms	61
1.4	Characterizations	68
1.4.1	Regular path characterization	68
1.4.2	Logical characterization	72
1.4.3	Handle replacement	79
1.4.4	Graph expressions	81
1.5	Recognition	82
	References	88
2	Hyperedge Replacement Graph Grammars	95
	(F. Drewes, H.-J. Kreowski, A. Habel)	95
2.1	Introduction	97
2.2	Hyperedge replacement grammars	100
2.2.1	Hypergraphs	102
2.2.2	Hyperedge replacement	104
2.2.3	Hyperedge replacement derivations, grammars, and lan- guages	105
2.2.4	Bibliographic notes	109
2.3	A context-freeness lemma	111
2.3.1	Context freeness	111
2.3.2	Derivation trees	114
2.3.3	Bibliographic notes	115
2.4	Structural properties	116
2.4.1	A fixed-point theorem	116

2.4.2 A pumping lemma 118

2.4.3 Parikh’s theorem 122

2.4.4 Bibliographic notes 123

2.5 Generative power 124

2.5.1 Graph-generating hyperedge replacement grammars . . 124

2.5.2 String-generating hyperedge replacement grammars . . 125

2.5.3 Further results and bibliographic notes 130

2.6 Decision problems 132

2.6.1 Compatible properties 132

2.6.2 Compatible functions 135

2.6.3 Further results and bibliographic notes 138

2.7 The membership problem 141

2.7.1 NP-completeness 141

2.7.2 Two polynomial algorithms 145

2.7.3 Further results and bibliographic notes 154

2.8 Conclusion 155

References 156

3 Algebraic Approaches to Graph Transformation - Part I: Basic Concepts and Double Pushout Approach 163

(A. Corradini, U. Montanari, F. Rossi, H. Ehrig, R. Heckel, M. Löwe) **163**

3.1 Introduction 165

3.2 Overview of the Algebraic Approaches 168

3.2.1 Graphs, Productions and Derivations 168

3.2.2 Independence and Parallelism 172

Interleaving 172

Explicit Parallelism 174

3.2.3 Embedding of Derivations and Derived Productions . . 176

3.2.4 Amalgamation and Distribution 178

Amalgamation 178

Distribution 179

3.2.5 Further Problems and Results 180

Semantics 180

Control 181

Structuring 182

Analysis 182

More general structures 182

3.3 Graph Transformation Based on the DPO Construction 182

3.4 Independence and Parallelism in the DPO approach 191

3.5 Models of Computation in the DPO Approach 200

3.5.1	The Concrete and Truly Concurrent Models of Computation	201
3.5.2	Requirements for capturing representation independence.	208
3.5.3	Towards an equivalence for representation independence.	212
3.5.4	The abstract models of computation for a grammar.	217
3.6	Embedding, Amalgamation and Distribution in the DPO approach	220
3.6.1	Embedding of Derivations and Derived Productions	220
3.6.2	Amalgamation and Distribution	224
	Amalgamation	224
	Distribution	225
3.7	Conclusion	228
3.8	Appendix A: On commutativity of coproducts	228
3.9	Appendix B: Proof of main results of Section 3.5	232
	References	240
4	Algebraic Approaches to Graph Transformation - Part II: Single Pushout Approach and Comparison with Double Pushout Approach	247
	(H. Ehrig, R. Heckel, M. Korff, M. Löwe, L. Ribeiro, A. Wagner, A. Corradini)	247
4.1	Introduction	249
4.2	Graph Transformation Based on the SPO Construction	250
4.2.1	Graph Grammars and Derivations in the SPO Approach	250
4.2.2	Historical Roots of the SPO Approach	258
4.3	Main Results in the SPO Approach	259
4.3.1	Parallelism	260
	Interleaving	260
	Explicit Parallelism	264
4.3.2	Embedding of Derivations and Derived Productions	268
4.3.3	Amalgamation and Distribution	273
4.4	Application Conditions in the SPO Approach	278
4.4.1	Negative Application Conditions	278
4.4.2	Independence and Parallelism of Conditional Derivations	282
	Interleaving	282
	Explicit Parallelism	284
4.5	Transformation of More General Structures in the SPO Approach	287
4.5.1	Attributed Graphs	288
4.5.2	Graph Structures and Generalized Graph Structures	292

4.5.3	High-Level Replacement Systems	294
4.6	Comparison of DPO and SPO Approach	296
4.6.1	Graphs, Productions, and Derivations	298
4.6.2	Independence and Parallelism	301
	Interleaving	301
	Explicit Parallelism	302
4.6.3	Embedding of Derivations and Derived Productions . .	305
4.6.4	Amalgamation and Distribution	306
4.7	Conclusion	308
	References	309
5	The Expression of Graph Properties and Graph Transformations in Monadic Second-Order Logic	313
	(B. Courcelle)	313
5.1	Introduction	315
5.2	Relational structures and logical languages	317
5.2.1	Structures	317
5.2.2	First-order logic	318
5.2.3	Second-order logic	319
5.2.4	Monadic second-order logic	320
5.2.5	Decidability questions	322
5.2.6	Some tools for constructing formulas	324
5.2.7	Transitive closure and path properties	327
5.2.8	Monadic second-order logic without individual variables	331
5.2.9	A worked example: the definition of square grids in MS logic	332
5.3	Representations of partial orders, graphs and hypergraphs by relational structures	334
5.3.1	Partial orders	334
5.3.2	Edge set quantifications	335
5.3.3	Hypergraphs	339
5.4	The expressive powers of monadic-second order languages . .	340
5.4.1	Cardinality predicates	340
5.4.2	Linearly ordered structures	342
5.4.3	Finiteness	344
5.5	Monadic second-order definable transductions	346
5.5.1	Transductions of relational structures	346
5.5.2	The fundamental property of definable transductions . .	351
5.5.3	Comparisons of representations of partial orders, graphs and hypergraphs by relational structures	354

5.6 Equational sets of graphs and hypergraphs 356

5.6.1 Equational sets 356

5.6.2 Graphs with ports and VR sets of graphs 362

5.6.3 Hypergraphs with sources and HR sets of hypergraphs . 367

5.7 Inductive computations and recognizability 372

5.7.1 Inductive sets of predicates and recognizable sets 372

5.7.2 Inductivity of monadic second-order predicates 379

5.7.3 Inductively computable functions and a generalization of Parikh’s theorem 383

5.7.4 Logical characterizations of recognizability 389

5.8 Forbidden configurations 390

5.8.1 Minors 390

5.8.2 The structure of sets of graphs having decidable monadic theories 396

References 397

6 2-Structures - A Framework For Decomposition And Transformation Of Graphs **401**

(A. Ehrenfeucht, T. Harju, G. Rozenberg) **401**

6.1 Introduction 403

6.2 2-Structures and Their Clans 404

6.2.1 Definition of a 2-structure 404

6.2.2 Clans 407

6.2.3 Basic properties of clans 410

6.3 Decompositions of 2-Structures 411

6.3.1 Prime clans 411

6.3.2 Quotients 412

6.3.3 Maximal prime clans 416

6.3.4 Special 2-structures 418

6.3.5 The clan decomposition theorem 419

6.3.6 The shape of a 2-structure 421

6.3.7 Constructions of clans and prime clans 424

6.3.8 Clans and sibas 427

6.4 Primitive 2-Structures 430

6.4.1 Hereditary properties 430

6.4.2 Uniformly non-primitive 2-structures 433

6.5 Angular 2-structures and T-structures 434

6.5.1 Angular 2-structures 434

6.5.2 T-structures and texts 438

6.6 Labeled 2-Structures 442

6.6.1	Definition of a labeled 2-structure	442
6.6.2	Substructures, clans and quotients	444
6.7	Dynamic Labeled 2-Structures	447
6.7.1	Motivation	447
6.7.2	Group labeled 2-structures	449
6.7.3	Dynamic labeled 2-structures	450
6.7.4	Clans of a dynamic Δ^δ 2-structure	454
6.7.5	Horizons	455
6.8	Dynamic ℓ 2-structures with Variable Domains	457
6.8.1	Disjoint union of ℓ 2-structures	457
6.8.2	Comparison with grammatical substitution	458
6.8.3	Amalgamated union	459
6.9	Quotients and Plane Trees	460
6.9.1	Quotients of dynamic Δ^δ 2-structures	460
6.9.2	Plane trees	461
6.10	Invariants	469
6.10.1	Introduction to invariants	469
6.10.2	Free invariants	470
6.10.3	Basic properties of free invariants	472
6.10.4	Invariants on abelian groups	473
6.10.5	Clans and invariants	476
	References	476
7	Programmed Graph Replacement Systems	479
	(A. Schürr)	479
7.1	Introduction	481
7.1.1	Programmed Graph Replacement Systems in Practice	481
7.1.2	Programmed Graph Replacement Systems in Theory	482
7.1.3	Contents of the Contribution	483
7.2	Logic-Based Structure Replacement Systems	484
7.2.1	Structure Schemata and Schema Consistent Structures	485
7.2.2	Substructures with Additional Constraints	492
7.2.3	Schema Preserving Structure Replacement	498
7.2.4	Summary	504
7.3	Programmed Structure Replacement Systems	505
7.3.1	Requirements for Rule Controlling Programs	506
7.3.2	Basic Control Flow Operators	507
7.3.3	Preliminary Definitions	510
7.3.4	A Fixpoint Semantics for Transactions	512
7.3.5	Summary	518

7.4	Context-sensitive Graph Replacement Systems – Overview . . .	519
7.4.1	Context-sensitive Graph Replacement Rules	520
7.4.2	Embedding Rules and Path Expressions	523
7.4.3	Positive and Negative Application Conditions	526
7.4.4	Normal and Derived Attributes	527
7.4.5	Summary	530
7.5	Programmed Graph Replacement Systems – Overview	530
7.5.1	Declarative Rule Regulation Mechanisms	532
7.5.2	Programming with Imperative Control Structures . . .	534
7.5.3	Programming with Control Flow Graphs	537
7.5.4	Summary	540
	References	541
	Index	547