Foundations and Interpretation of Quantum Mechanics

*In the Light of a
Critical-Historical Analysis
of the Problems and of a
Synthesis of the Results*

With a Foreword by Giorgio Parisi

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Critical-Historical Analysis
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Synthesis of the Results*

With a Foreword by Giorgio Parisi

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Alla vita che cresce
e si impone intorno a me

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*History is the most fundamental science,
for there is no human knowledge
which cannot loose its scientific character
when men forget the conditions under which it originated,
the question to which it answered,
and the functions it was created to serve.*

*A great part of the mysticism
and superstition of educated men
consists of knowledge which has broken loose
from its historical moorings*

Benjamin FARRINGTON

quoted in [SCHRÖDINGER 1952, : 479]

*The first process, therefore, in the effectual study of the sciences,
must be one of simplification and reduction
of the results of previous investigations
to a form in which the mind can grasp them*

J. C. MAXWELL

quoted in [JAUCH 1968, : 67]

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FOREWORD

Quantum mechanics is one of the greatest intellectual achievements of this century. As an effect of its discovery, the very concept of physical reality was changed, the words ‘observation’, ‘measurement’, ‘prediction’, ‘state of the system’ acquired a new and deeper meaning.

Probability was not unknown in physics: it was introduced by Boltzmann in order to control the behaviour of a system with a very large number of particles. It was the missing concept in order to understand the thermodynamics of macroscopic bodies, but the structure of the physical laws remained still deterministic. The introduction of probability was needed as a consequence of our lack of knowledge of the initial conditions of the system and of our inability to solve an enormous number of coupled non linear differential equations. If we were both infinitely able experimentalists and infinitely able mathematicians, probability would be useless in classical physics: it is only a tool which allows imperfect beings, with a bounded brain like us, to control the behaviour of many particle systems.

In quantum mechanics, the tune is different: if we have 10^6 radioactive atoms *no* intrinsic unknown variables decide which of them will decay firstly. What we observe experimentally seems to be an irreducible random process. The original explanation of this phenomenon in quantum mechanics was rather unexpected. All atoms have the same probability of having decayed: only when we observe the system we select which atoms have decayed in the past. In spite of the fact that this solution seems to be in contrast with common sense, it is the only possible one in the framework of the conventional interpretation of quantum mechanics. The old problem of the relations among the observer and the observed object, discussed for centuries by philosophers, had a unexpected evolution and now it must be seen from a new, completely different perspective.

It is remarkable that all these conceptual changes were not due to reflections aimed to understanding better the *philosophical* problems, but came out naturally as a byproduct of the effort to explain the properties of the radiation emitted and absorbed in various conditions, particularly the spectrum of the hydrogen atom. Heisenberg, de Broglie, Pauli, Dirac and many others invented a formalism that was able to explain and predict the experimental data and this formalism led, beyond the very intention of the men who constructed it, to this conceptual revolution.

Once established, quantum mechanics became a wonderful and extremely powerful tool. The properties of the different materials, the whole chemistry, became for the first time objects that could be predicted from the theory and not only phenomenological rules deduced from experiments. The technological discovery that shaped the second half of this century, the transistor (i.e. the basis of all the modern electronics and computers) could not be invented without a deep command of quantum mechanics.

In spite of this remarkable success, quantum mechanics remains mysterious. It is not only the problem of explaining its meaning without using advanced mathematics that forbids a simple exposition of its properties to the layman. The rules are weird: the fundamental objects are complex amplitudes and probabilities are the modulus square. A scientist like Feynman, who

contributed to a new formulation of quantum mechanics and made some of the crucial steps to extend quantum mechanics into the relativistic domain, wrote once *nobody understands quantum mechanics*. Also Poliakov, one of the greatest living theoretical physicists said in a lecture that *eventually someone has to explain why the probability is the modulus square of a complex amplitude*.

These statements may seem rather strange: each year quantum mechanics is taught in thousands of university courses, computations based on quantum mechanics are routinely compared to experiments and found to be correct with incredible precision. However, the meaning of Feynman's and Poliakov's statements becomes more clear if we ask the professors of quantum mechanics about the meaning, the interpretation of quantum mechanics. If we don't ask which are the rules for predicting the output of an experiment, but which is the *nature* of the external world, which is the *meaning* (if any) of an objective physical reality, what happens before, during and after measurements, the opinions become quite diverse and some dissatisfaction is often present in our answers. Somebody may even tell you: *I do not believe that the world is done in this way, but the computations turns out to be accurate*.

Of course we could take the position that the only thing we can know is a set of rules (i.e. quantum mechanics) to deduce the results of the experiments and any further question is only a metaphysical irrelevant question which does not belong to the field of physics. However this answer would be considered unsatisfactory by most physicists. In trying to give an answer to these questions, many different schools arose: in standard books one usually finds the *classic* Copenhagen interpretation and sometimes the puzzling many worlds interpretation, which brings the positions of the Copenhagen school to a logic but paradoxical conclusion.

Many different interpretations have been proposed, starting from de Broglie's pilot wave and arriving to the most recent stochastic quantum mechanics or to the consistent history approach. These different and quite diverse approaches are normally ignored in the manuals of quantum mechanics, but they are very interesting and viable alternatives to the usual Copenhagen interpretation.

The progresses in recent years have not only concentrated on the problems of interpretation that could be (wrongly) dismissed by some people as metaphysical, because they are beyond experimental tests. In the last twenty years, the whole complex of problems connected to quantum mechanics and the meaning of measurements started to be studied from a new perspective. Real, not only *Gedanken* experiments began to be done on some of the most elusive properties of quantum mechanics, i.e. the existence of correlations among spatially separated systems that could not be explained using the traditional concept of probability. The precise quantum mechanical meaning of measurements started to be analyzed in a more refined way (e.g. quantum non demolition measurements were introduced) and various concepts from statistical mechanics and other fields of physics began to be used.

This is not only an academic or philosophical problem. The possibility of constructing a quantum computer, that would improve the speed of present day computers by an incredible factor, is deeply rooted in these achievements. It is now clear that a quantum computer can solve problems, which on conventional computers take a time exploding as exponent of some parameter (e.g. the factorization into primes of a number of length N), in a time which is only a polynomial in N . The technical problems to be overcome for constructing a quantum computers are not easy to solve, but this result has an high conceptual status, telling us how deeply quantum mechanics differs from classical mechanics. Another possible practical application of these developments is quantum cryptography, in which a message is transmitted in such a way that it cannot be read without interfering with it. Another quantum-information puzzling phenomenon,

the teleportation, has been recently proved experimentally to exist and it is a very active area of experimental research.

All these aspects of quantum mechanics form a gigantic field in which a very large amount of work was done, but unfortunately this rich and deep activity remains confined to specialized reviews and unknown to most physicists. Of course many books present these new progresses, but they deal with only a part of the area. In this unique book, Gennaro Auletta reviews and summarizes the main achievements of the whole field, not only a part of it. One of the aims of the author was in fact completeness, in the sense of bringing to the attention of the reader all the different points of view that have been presented till now, discussing their relative advantages and disadvantages. The immense bibliography quoted in the book (which can be found at the end) is a proof of the attention which the author has devoted also to many interesting, but forgotten papers. In this way we obtain a very rich, deep and precise picture of the status of the art which looks like a mosaic. Such a comprehensive presentation was lacking till now and this book fills very well this lacuna. At the end of the book the author presents his original synthesis of the problem of the foundation of quantum mechanics and a deep personal outline of the interpretation of quantum mechanics.

The book is written in a condensed, but clear style, where not only mathematical objects, but also the basic interpretative concepts are well defined, avoiding (as far as possible) statements which are unclear or open to many interpretations. This quest for precision and clarity, which is important everywhere, is crucial in this domain, especially in all problems connected to the foundation and the interpretation of quantum mechanics, in which many non-mathematical considerations must be done. One could arrive to wrong conclusions if one does not proceed in a careful way. In order to avoid to fall in such a trap the author has followed some strict methodological principles which are stated at the beginning of the book (e.g. *Do not infer ontological conclusions from formal premisses without specific and extraformal motivations*). The philosophical skills of the author helped him in this difficult task and also for this reason this book is quite different from any other published on the subject.

Giorgio Parisi

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