QUANTITATIVE GEOCHEMISTRY

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*In loving memory of my father,
Guilin Zou*

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Preface

Modern geochemistry utilizes three powerful tools: (major and trace) elements, isotopes, and equations, to study various Earth and environmental processes. A combination of the experimental tools (elements and isotopes) with theoretical tools (equations) provides penetrating insights into the Earth and environmental processes. The aim of this book is to link equations more closely with geochemical measurements, including elemental abundances and (radiogenic, radioactive and stable) isotopic compositions. The importance to use equations in scientific research has been best stated by Albert Einstein, "Equations are more important to me, because politics is for the present, but *an equation is something for eternity*."

The book covers many exciting aspects of geochemistry where equations have been useful. Chapters 1 to 5 introduce the behaviors of trace elements and uranium-series isotope disequilibria during partial melting. Chapter 6 describes the behaviors of trace elements during crystallization, assimilation and mixing. All the above 6 chapters deal with forward modeling of geochemical processes while Chapter 7 introduces inverse geochemical modeling. Chapters 8 and 9 describe the treatments of geochemical data using error analysis and least-square fitting. These treatments are applicable not only to geochemical data but also to experimental data in other scientific fields. Chapters 10 and 11 are concerned with mass spectrometry (for the measurement of isotopic compositions using mass spectrometers) and clean lab chemistry, with emphasis on mass fractionation in ionization processes and single and double spike isotope dilution methods. Chapter 12 provides some fun in Pb isotope modeling from an uncommon perspective. For example, the

well-known zircon Concordia plots are treated with the second derivatives. Chapter 13 describes geochemical kinematics and dynamics and emphasizes the combination of geochemistry with physics.

Although the mathematical methods in this book include algebra, calculus, differential equations, matrix, statistics, and numerical analyses, students with background in algebra and calculus alone are able to understand most of the contents. In addition, since simple models are presented before more complex models and additional parameters are added gradually, students should not worry about the difficulties in mathematics.

The book is problem-oriented, and thus does not cover everything in geochemistry. It is hoped that the book may help enhance students' abilities to understand, criticize, appreciate, and apply available models for deep insights of fundamental geochemical problems. For some students inspired to explore further, they may develop creative models of their own, on the basis of a full understanding of the model development in this book first.

In addition to the author's published and unpublished theoretical work, the author heavily drew upon the quantitative models of more than a hundred scientists listed in the references. I hope that their scientific contributions are correctly acknowledged in the text, and I wish to express my sincere thanks to all of them. Note that many equations in geochemistry were originally derived by geochemists (in a broad sense) themselves, not simply borrowed from chemistry, physics, or engineering literature.

It is also a great pleasure to thank many people individually for their help. First and foremost, I would like to express my deepest gratitude to Kevin D. McKeegan for his enduring inspiration, support, generosity, and mentoring. Sincere thanks also go to Mark T. Harrison for his valuable comments and corrections on the manuscript and to Jerry Wasserburg for his encouragement "keep doing science". I have greatly benefited from interactions with An Yin, John Wasson, Fred Frey, Alan Zindler, Mary Reid, Asish Basu, Vincent Salters, Roy Odom, David D. Loper, Sam Mukasa, Craig Manning, Ken Sims and late Denis M. Shaw. I learned from An Yin and John Wasson to add physics to geochemical

research. I am grateful to Asish Basu for bringing me to USA for graduate studies 13 years ago. John Hora generously provided the fantastic cover image of the Parinacota volcano. I am thankful to Hongwu Xu, Huifang Xu, Ying Ma, Mariana Cosarinsky, and Yingfeng Xu for their help with pictures and references. I am indebted to Chee-Hok Lim for efficient editing, to Jimmy Low for excellent cover design, and to Imperial College Press Chairman Kok Khoo Phua for book invitation. Last, but not the least, I thank my wife Yuhong Wang and my daughter Joy Zou for their support and encouragement, without which this book would not have been completed.

I'd deeply appreciate hearing about any errors, criticisms, suggestions, difficulties encountered, and perhaps gentle word of encouragement. The author's email address is hzou@ess.ucla.edu. Additional supporting materials related to this book will be posted on the author's home page: <http://www2.ess.ucla.edu/~hzou>.

Haibo Zou

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