Selected Topics on Continuous-Time Controlled Markov Chains and Markov Games

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ICP Advanced Texts in Mathematics – Vol. 5

Selected Topics on Continuous-Time Controlled Markov Chains and Markov Games

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SELECTED TOPICS ON CONTINUOUS-TIME CONTROLLED
MARKOV CHAINS AND MARKOV GAMES

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To Guadalupe and my parents, Emilio and Michèle.

To Marina, Gerardo, Adrián, Claudia, and Lucas.

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Preface

This book concerns continuous-time controlled Markov chains and Markov games. The former, which are also known as continuous-time Markov decision processes, form a class of stochastic control problems in which a single decision-maker wishes to optimize a given objective function. In contrast, in a Markov game there are two or more decision-makers (or players, or controllers) each one trying to optimize his/her own objective function.

The main features of the control and game models studied in the book are that the time variable is *continuous*, the state space is *denumerable*, and the control (or action) sets are *Borel spaces*. Moreover, the transition and reward rates of the dynamical system may be *unbounded*. Controlled Markov chains and Markov games have many important applications in areas such as telecommunication networks, population and epidemic models, engineering, operations research, etc. Some of these applications are illustrated in this book.

We note that most of the material presented here is quite recent: it has been published in the last six years, and it appears in book form for the first time.

One of the main goals of this book is to study the so-called advanced optimality criteria for controlled Markov chains (e.g., bias, overtaking, sensitive discount, and Blackwell optimality), which are refinements of the basic criteria, namely, discounted and average reward optimality. To make this a self-contained book, we also give the main results on the existence of controlled Markov chains and the basic optimality criteria. For the corresponding technical details — some of which have been skipped here — the reader can consult Guo and Hernández-Lerma's *Continuous-Time Markov Decision Processes: Theory and Applications* [52].

A particular emphasis is made regarding the application of the results

presented in the book. One of our main concerns is to propose assumptions on the control and game models that are easily verifiable (and verified) in practice. Furthermore, we study an algorithm to solve a certain class of control models, and establish some approximation results that allow us to give precise numerical approximations of the solutions to some problems of practical interest.

Hence, the book has an adequate balance between, on the one hand, theoretical results and, on the other hand, applications and computational issues. It is worth mentioning that the latter were, somehow, missing in the literature on continuous-time controlled Markov chains.

Finally, the topic of zero-sum two-person continuous-time Markov games, for both the basic — discounted and average payoff — and some “advanced” optimality criteria — bias and overtaking equilibria — appears for the first time in book form.

This book is mainly addressed to researchers in the fields of stochastic control and stochastic games. Indeed, it provides an extensive, rigorous, and up-to-date analysis of continuous-time controlled Markov chains and Markov games. It is also addressed to advanced undergraduate and beginning graduate students because the reader is not supposed to have a high mathematical background. In fact, a working knowledge of calculus, linear algebra, probability, and continuous-time Markov chains (at the level of, say, Chapter 4 of R. Durrett’s book *Essentials of Stochastic Processes* [31]) should suffice to understand the material herein. As already mentioned, the reader interested in the theoretical foundations of controlled Markov chains can consult [52].

We have carefully written this book, with great dedication and commitment. We apologize, however, for any errors and omissions it might contain.

The authors, April 2011

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