NEW TRENDS IN GEOMETRY

THEIR ROLE IN THE NATURAL AND LIFE SCIENCES

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Claudio Bartocci

Università di Genova, Italy

Luciano Boi

École des Hautes Études en Sciences Sociales, France

Corrado Sinigaglia

Università degli Studi di Milan, Italy

Editors

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CONTENTS

Preface by <i>Claudio Bartocci, Luciano Boi and Corrado Sinigaglia</i>	vii
Part I: Geometry, Theoretical Physics and Cosmology	1
Chapter 1: Claudio Bartocci and Ugo Bruzzo, <i>The Emergence of Algebraic Geometry in Contemporary Physics</i>	3
Chapter 2: Mauro Carfora, <i>Quantum Gravity and Quantum Geometry</i>	17
Chapter 3: Ugo Moschella, <i>The de Sitter and Anti-de Sitter Universes</i>	35
Chapter 4: Jean-Pierre Luminet, <i>Geometry and Topology in Relativistic Cosmology</i>	81
Part II: The Problem of Space in Neurosciences	105
Chapter 5: Leonardo Fogassi, <i>Space Coding in the Cerebral Cortex</i>	107
Chapter 6: Anna Berti and Alessia Folegatti, <i>Action and Space Representation</i>	127
Chapter 7: Claudio Brozzoli and Alessandro Farnè, <i>The Space Representations in the Brain</i>	137
Chapter 8: Corrado Sinigaglia and Chiara Brozzo, <i>The Enactive Constitution of Space</i>	157
Part III: Geometrical Methods in the Biological Sciences	171
Chapter 9: Francis Bailly and Giuseppe Longo, <i>Causes and Symmetries in Natural Sciences: The Continuum and the Discrete in Mathematical Modelling</i>	173
Chapter 10: Riccardo Broglia, <i>Topological Invariants of Geometrical Surfaces and the Protein Folding Problem</i>	211

Chapter 11: Jean-François Sadoc, <i>The Geometry of Dense Packing and Biological Structures</i>	221
Chapter 12: Luciano Boi, <i>When Topology and Biology Meet 'For Life': The Interactions Between Topological Forms and Biological Functions</i>	243
About the Contributors	307
Index	313

Preface

C. Bartocci, L. Boi, C. Sinigaglia (Eds.)

1. Scope and Aims of the Book

There are two major motivations for proposing this book. The first is the conviction that the integration of mathematics and physics, molecular and cell biology, and neurosciences will constitute the new frontier and challenge for twenty-first century science. The second relates to the fact that the exciting and appealing science in the twenty-first century is likely to evolve among, not within, traditional disciplines. Therefore, the book and *More geometrico* project focus on the interactions between mathematics, physics, biology and neurosciences, and are aimed at exploring new geometrical and topological modelling in a variety of physical, biological and neuroscience fields.

The authors concentrate on some new, extremely valuable interfaces of mathematical methods and modelling with the physical and life sciences. Its major aim is to further our understanding of the multilevel and scale-change phenomena in these disciplines. One of the main goals of the various contributions is to study the central role of a multilevel and scale-change approach in different fields such as neuroscience, systems biology, and quantum physics and geometry.

We think it is more and more important to inject ideas and methods from modern differential geometry and algebraic topology into neuroscience and molecular and cellular biology and to inspire new directions in these mathematical fields from discussions on the major problems in macroscopic physics and the biological sciences.

The contributions are aimed at giving a precise idea of some far-reaching connections between mathematics, physics, biology and cognition. Its essential scope is to develop rigorous interdisciplinary and integrative research. One important goal is to show that several methods and techniques, especially from algebraic topology and differential geometry, are profoundly involved at different scales and various levels of organisations in the physical and biological processes. Various contributions emphasise the urgent need for developing new mathematical methods, models and techniques suited to work out a mathematical dynamical theory of the emergence of natural and living patterns and behaviours. For example, in

our view, one particularly interesting task would consist of explaining to what extent the mathematical structure and spatial-temporal events that constitute the natural frame of living organisms may influence their bio-chemical, physiological and cognitive organisation. In this perspective, the most important and complex work to be carried out is to construct both qualitative and quantitative mathematical and physical models capable of describing and explaining the effective dynamics, properties and invariants of phenomena at different levels of organisation and action. This is, we think, one of the most important conceptual and philosophical challenges of today's interdisciplinary research.

2. Three Fundamental Themes

2.1. *Mathematical modelling in the neurosciences*

One important question raised in the book is the following: what differentiates the topological and metric structures of the 'physical' space, which is embodied into (or internalised by) the neurophysiological space in the brain, from the properties of the perceptive and cognitive space obtained very likely by deforming the first according to certain general (mathematical and/or physical) laws? The answer to this crucial questions is at the core of the present research in the neurosciences, the major issues of which are: (i) The mechanisms by which our complex sensorial systems such as the visual, the sensorimotor and the vestibular systems drive our perception of the surrounding space and the movements required in order to reach it. (ii) The perception of movement and its neurophysiological basis, and the role of action in the perception of the third dimension and in the cognitive grasping of the properties and qualities of objects localised in our (near and far) space. (iii) The mathematical and physical grounds of human cognition and, reciprocally, the biological and cognitive roots of our mathematical skills.

2.2. *Geometrical and topological methods in the life sciences*

A fundamental goal of the contributions in this field is to show that there are effective models and techniques from mathematical sciences, which can be used to describe many fundamental properties and behaviours observed in biological systems. More precisely, an important part of the present research is aimed at demonstrating that the complex topology and dynamics of DNA-proteins complexes are closely linked to the multilevel epigenetic regulation and to the cell's spatial and functional organisation. It has been emphasised that the geometrical structure and topological form of nuclear components (DNA, nucleosome, chromatin, chromosome etc.) play an important role in the cell differentiation and organism growth. For example, at the molecular and supramolecular level, enzyme topoisomerases, which convert DNA from one topological form to another, appear to have a profound role in the central

genetic events of replication, transcription, recombination and repair. Second, certain topological mechanisms are involved in the fundamental biological process of the compaction of chromatin into the chromosome during the interphase and the metaphase. Third, new mathematical methods and models have been suggested relating to the cells' differentiation and their complex spatial organisation during the different phases of the development of the embryos. It appears an essential task to provide mathematical models and techniques which are more global and dynamic, in order to be able to rethink successfully the causal connection between form and function in the biological sciences.

2.3. *Open theoretical problems and mathematical perspectives in relativistic and quantum physics*

We think it is important to address the issue of the role of algebro-geometric and topological methods in the developments of quantum physics, especially in gauge theory as well as string theory, and tried to show how these methods can provide a deeper understanding of physical phenomena at different scales. We consider that the unification of gravitation and quantum physics requires some fundamental breakthroughs in our understanding of the relationship between space-time and quantum processes. In particular, the superstring theories lead to guessing that the usual structure of space-time at the Planck scale must be dropped from physical thought. A very interesting hypothesis, which is discussed in some chapters of this book (in Part I), is that the global geometrical properties of the manifold model of space-time (either Lorentzian or Riemannian) play a major role in quantum field theories and that, consequently, several physical quantum effects arise from the non-local metrical and topological structures of these manifolds. Modern Kaluza–Klein theories, superstring theory and, in a different way, quantum gravity and non-commutative geometry, showed that space-time symmetries and physical symmetries might be unified through the introduction of new structures (dimensions, invariants, supersymmetries) of space with a different topology. That essentially means that ‘hidden’ symmetries of fundamental physics can be related to the phenomenon of topological change of certain classes of non-smooth manifolds.

3. Brief Conclusions

We hope this book will contribute to showing the increasingly fundamental role played by geometrical and topological ideas and methods in several relevant fields of research relating to a number of important recent developments in the natural and life sciences. In fact, it is more and more emerging that some geometrical and topological concepts could help to describe and explain a variety of amazing structures and behaviours characterising physical reality, living systems and human

cognition. Indeed, many processes and organisational principles in these fields seem to be closely related to the invariant action of certain fundamental geometrical and topological objects and structures.

Let us conclude by stressing once again the very purpose of this book, which is to explore and understand some far-reaching interfaces where geometry, topology, physics, dynamics, biology and neurosciences seem to interact profoundly, in a great and significant overlap of knowledge among mathematicians, physicists, biologists, neurophysiologists and philosophers of science. Last but not least, one unifying theme characterises this book: Geometry may best be interpreted as a way of thinking rather than a mere formal language or a collection of specific subject areas. There is, perhaps, no branch of mathematics that cannot be considered a part of geometry, and there is, most likely, no field in the natural and life sciences unrelated to the enlightening influence of geometry, when approached in the right, open spirit. Resting upon some geometric-minded theories and methods, the authors propose various insights and prospects for future research. We expect they will contribute to drawing a more comprehensive and meaningful landscape of natural and living phenomena.

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