Finite Element Modeling of Multiscale Transport Phenomena

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Preface

For more than five decades finite element method has been regarded as the foremost numerical technique for the solution of governing equations of engineering problems. Using this method powerful computer schemes, which combine mathematical rigor with geometrical flexibility in dealing with complex problems, have been developed. These schemes are, in general, the most reliable predictive simulation tools available to researchers and design engineers. Inherent flexibility and sound mathematical basis of the technique has been the main reason for its progress and evolution over the years, enabling numerical analysts to use the technique to solve a wide variety of problems. Recent advances in the power and capability of computer systems has made simulation of some of the most complex physical situations a realistic possibility. However, novel concepts need to be incorporated into existing numerical schemes in order to take advantage of recent advances in low cost computing. In this respect, the solution of multiscale phenomena, in which significant and rapid variations in the behavior of field unknowns either precludes the application of traditional techniques or renders their use complicated and unyielding, has been the subject of intense research during the past decade. During this period many important research papers have appeared which provide robust mathematical foundations for the construction of practical finite element schemes for multiscale problems.

The main focus of this book is to provide a simple to follow account of the development of a class of practical multiscale weighted residual finite element schemes for field problems encountered in fluid flow and transport processes. In particular, dealing with the generic multiscale phenomena which affect the design and analysis of chemical engineering and polymer processing operations has been our objective.

The book starts with an explanation of the weighted residuals finite element technique to provide the necessary background for the discussions presented later on in the book. Readers who have not previously used weighted

residuals finite element schemes should, nevertheless, be able to follow the discussions presented in chapters dealing with the extension of this technique to multiscale problems. Almost all of the topics introduced in the text have been supplemented with solved examples. These examples can be used as a guide by readers to apply the constructed schemes to their own problems or they may use the described methodology for the development of multiscale schemes applicable to other problems.

Finally, we have included a detailed listing of the computer code used to solve many of the examples given in this book and provided sample input and output files. Readers can repeat the illustrated examples and gain experience for extending the program to perform their own multiscale finite element simulations.

Vahid Nassehi
Mahmoud Parvazinia

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