Labelled Markov Processes

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Prakash Panangaden
McGill University, Canada

Published by

Imperial College Press
57 Shelton Street
Covent Garden
London WC2H 9HE

Distributed by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN-13 978-1-84816-287-7

ISBN-10 1-84816-287-1

Printed in Singapore.

For my father, John Anthony Panangaden, and the memory of
my mother, Mary Atokaren.

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Preface

This short book consists of two parts. The first part is an exposition of basic measure theory and probability theory on continuous state spaces. It does not replace the many excellent texts on the subject but it gives a condensed and, I hope, still readable account suitable for graduate students in computer science with a background in concurrency and semantics and without the standard undergraduate analysis curriculum. I learned this subject when I began my research into labelled Markov processes and this part of the book represents what I found useful at that time.

It is based in part on lectures that I gave at Aarhus University in 1997 and at a summer school in Udine later that same year. I had intended to let them serve as stand alone lectures but after giving a version of these lectures at Oxford in 2003–04, I was persuaded to turn them – together with an account of recent research – into book form. In the end I felt it hopeless to keep pace with the vast outpouring of papers in the last few years and have described only the work with which I was directly involved. I have, in the last chapter, given some pointers to the work of others. I feel that a comprehensive account of the whole field of probabilistic verification and related topics such as approximation and connections to planning, machine learning and performance evaluation would be premature at this point. Perhaps in the future, some more ambitious person will attempt it when the field has stabilised more.

I would like to thank Mogens Nielsen for encouraging me to teach a course on this topic at the University of Aarhus during my sabbatical year 1996–97 and Furio Honsell, Eugenio Moggi and Catuscia Palamidessi for inviting me to lecture on this at a summer school in Udine in 1997. I have benefitted from interactions with several friends, colleagues and students (those are not disjoint sets): Samson Abramsky, Christel Baier, Rick Blute,

Pablo Castro, Alexandre Bouchard-Côte, Philippe Chaput, Vincent Danos, Luca de Alfaro, Erik de Vink, Luc Devroye, Ernst-Erich Doberkat, Devdatt Dubashi, Martin Escardo, Abbas Edalat, Norm Ferns, Vineet Gupta, Holger Hermanns, Jørgen Hoffmann-Jørgensen, Chris Hundt, Joost-Pieter Katoen, Dexter Kozen, Marta Kwiatkowska, François Laviolette, Keye Martin, Mike Mislove, Gethin Norman, Catuscia Palamidessi, Joelle Pineau, Gordon Plotkin, Doina Precup, Jan Rutten, Roberto Segala, Alex Simpson, Jonathan Taylor, Franck van Breugel, James Worrell but most of all Josée Desharnais and Radha Jagadeesan. I would like to thank Jane Panangaden for the cover drawing.

Prakash Panangaden
Montreal West
September 2008

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