Computability in Context

Computation and Logic in the Real World

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Preface

Computability has played a crucial role in mathematics and computer science – leading to the discovery, understanding and classification of decidable/undecidable problems, paving the way to the modern computer era and affecting deeply our view of the world. Recent new paradigms of computation, based on biological and physical models, address in a radically new way questions of efficiency and even challenge assumptions about the so-called Turing barrier.

This book addresses various aspects of the ways computability and theoretical computer science enable scientists and philosophers to deal with mathematical and real world issues, ranging through problems related to logic, mathematics, physical processes, real computation and learning theory. At the same time it focuses on different ways in which computability emerges from the real world, and how this affects our way of thinking about everyday computational issues.

But the title *Computability in Context* has been carefully chosen. The contributions to be found here are not strictly speaking ‘applied computability’. The literature directly addressing everyday computational questions has grown hugely since the days of Turing and the computer pioneers. The *Computability in Europe* conference series and association is built on the recognition of the complementary role that mathematics and fundamental science plays in progressing practical work; and, at the same time, of the vital importance of a sense of context of basic research. This book positions itself at the interface between applied and fundamental research, prioritising mathematical approaches to computational barriers.

For us, the conference *Computability in Europe 2007: Computation and Logic in the Real World* was a hugely exciting – and taxing – experience. It brought together a remarkable assembly of speakers, and a level of participation around issues of computability that would surely have astounded Turing and those other early pioneers of ‘computing with understanding’. All of the contributions here come from invited plenary speakers or Pro-

gramme Committee members of CiE 2007. Many of these articles are likely to become key contributions to the literature of computability and its real-world significance. The authors are all world leaders in their fields, all much in demand as speakers and writers. As editors, we very much appreciate their work.

Barry Cooper and Andrea Sorbi

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