Black Holes

An Introduction

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Published by

Imperial College Press
57 Shelton Street
Covent Garden
London WC2H 9HE

Distributed by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

First published 2005

Reprinted 2006

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ISBN 1-86094-586-4

ISBN 1-86094-588-0 (pbk)

Preface

New science can seem quite weird at first: Newton's mystical action-at-a-distance; Maxwell's immaterial oscillations in a vacuum; the dice-playing god of quantum mechanics. In due course however we come to accept how the world is and teach it to our students. Our acceptance, and theirs, comes principally through mastery of the hard details of calculations, not from generalised philosophical debate (although there is a place for that later).

Black holes certainly seem weird. We know they (almost certainly) exist, although no-one will ever 'see' one. And they appear to play an increasingly central role both in astrophysics and in our understanding of fundamental physics. It is now almost 90 years since the Schwarzschild solution was discovered, 80 years since the first investigations of the Schwarzschild horizon, 40 years on from the first singularity theorems and perhaps time that we can begin to dispel the weirdness and pass on something of what we understand of the details to our undergraduate students. This is what we attempt in this book. We have tried to focus on the aspects of black holes that we think are generally accessible to physics undergraduates who may not (or may) intend to study the subject further. Many of the calculations in this book can be done more simply using more sophisticated tools, but we wanted to avoid the investment of effort from those for whom these tools would be of no further use. Those who do go further will appreciate all the more the power of sophistication.

The presentation assumes a first acquaintance with general relativity, although we give a brief recapitulation of (some of) the main points in chapter 1. The treatment is however very incomplete: we do not consider the Einstein field equations because we do not demonstrate here that the black hole geometries are solutions of the equations. In fact, very little prior knowledge of relativity is required to study the properties of given black hole spacetimes. Chapter 2 is devoted to classical spherically symmetric, or non-rotating (Schwarzschild) black holes in the vacuum and chapter 3 to axially symmetric, or rotating (Kerr) ones. It is unfortunate that even simple calculations for Kerr black holes rapidly become algebraically complex. We have tried not to let this obscure the intriguing physics. We ask the reader to stick with it. After all, there may be a lot of it, but it is only elementary algebra. We give only a brief overview of charged black holes. These have played an important role as algebraically simpler models for many of the properties of rotating holes, and they are important as such in higher dimensions, but it is intrinsically difficult to maintain an interest in the

physics of objects that probably do not exist, especially since we are going to treat the rotating holes in detail anyway.

In chapter 4 we attempt to explain the quantum properties of black holes without recourse to quantum field theory proper. The calculations here are less rigorous than in the rest of the book (probably a gross understatement) but many variations on the standard theme are now available in textbooks, reviews and lecture notes on the internet and we see no merit in repeating these. We hope our approach is more useful than a crash course in quantum field theory. It does, of course, assume a more than superficial understanding of standard quantum theory. Chapter 5 closes the book with a brief review of black hole astrophysics in so far as it is relevant to the observation of black holes. We are aware that we have omitted many contemporary topics in black hole physics, not least the properties of general black holes, perturbation of black holes and the role of black holes in string theories. We regard these as beyond the scope of the book (and in the last case of the expertise of the authors). We hope (and believe) that working out long-hand the details of what we do include will provide a firm foundation for those students who will go on to study such advanced topics and a firm understanding and appreciation of the properties of black holes for those who do not.

Derek Raine
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