PHYSICS OF INTENSE CHARGED PARTICLE BEAMS IN HIGH ENERGY ACCELERATORS

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Ronald C Davidson
Hong Qin

Plasma Physics Laboratory and
Department of Astrophysical Sciences
Princeton University

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**FOR OUR CHILDREN
CYNTHIA, RONALD, JR., AND ANGELICA**

*In deep appreciation for the joy and love they
so generously share with family and friends.*

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Preface

Periodic focusing and cyclic accelerators, transport systems and storage rings have a wide range of applications ranging from basic scientific research in high energy and nuclear physics, to applications such as spallation neutron sources, heavy ion fusion, tritium production, and nuclear waste treatment. Of particular importance, at the high beam currents and charge densities of practical interest, are the effects of the intense self fields produced by the beam space charge and current on determining the detailed equilibrium, stability and transport properties, and the nonlinear dynamics of the system. Intense charged particle beams (or charge bunches), like one-component nonneutral plasmas, are a many-body collection of charged particles which exhibit a broad range of collective phenomena, such as plasma waves and instabilities. Moreover, the intense self fields in a charged particle beam can have a large influence on the detailed dynamics and stability behavior of the beam.

During the past decade, the common physics of one-component nonneutral plasmas and intense charged particle beams has become increasingly evident. Indeed, many important scientific advances have been made, both in terms of theoretical understanding and instrumentation techniques, since the properties of intense charged particle beams and laboratory-confined nonneutral plasmas were investigated using similar theoretical formalisms in *Physics of Nonneutral Plasmas* (Addison Wesley, 1990; reissued by World Scientific, 2001). Of course an intense charged particle beam (or charge bunch), when considered in the beam frame, is in fact a one-component nonneutral plasma, which would be expected to exhibit many collective properties similar to laboratory-confined pure ion plasmas or pure elec-

tron plasmas. In this regard, there is an extensive literature on the collective properties and nonlinear dynamics of laboratory-confined nonneutral plasmas which is relevant to the general subject matter of this book.

Physics of Intense Charged Particle Beams in High Energy Accelerators has been prepared as a graduate-level text which covers a broad range of topics related to the fundamental properties of the collective processes and nonlinear dynamics of intense charged particle beams. The subject matter is treated systematically from first principles using a unified theoretical approach, and the emphasis is on the development of basic concepts that illustrate the underlying physical processes in circumstances where intense self fields play a major role in determining the evolution of the system. The statistical models used to describe the properties of intense charged particle beams are based on the Vlasov-Maxwell equations, the macroscopic fluid-Maxwell equations, or the Klimontovich-Maxwell equations, as appropriate, and extensive use is made of theoretical techniques developed in the description of one-component nonneutral plasmas, and multispecies electrically-neutral plasmas, as well as established techniques in accelerator physics, classical mechanics, electrodynamics and statistical physics. These approaches are found to provide remarkably tractable and robust theoretical frameworks for describing the detailed nonlinear dynamics and collective processes in intense charged particle beams, including regimes where the intense self fields produced by the beam space charge and current play a major role in determining the detailed evolution of the system.

Physics of Intense Charged Particle Beams in High Energy Accelerators includes 75 problems and 71 figures and illustrations. Because of the book's emphasis on basic physics principles, and the thorough presentation format, it is intended to have a broad and lasting appeal to graduate students and researchers in the field.

Ronald C. Davidson
 Hong Qin
Princeton, New Jersey
 October, 2001

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The material for this book has evolved over more than one decade of teaching and research on the physics of intense charged particle beams and nonneutral plasmas. Many colleagues, students and collaborators in the accelerator physics and plasma physics communities have contributed significantly to the subject matter and selection of topics covered in this book. At the risk of overlooking key colleagues, we are particularly grateful for stimulating discussions with Roger Bangerter, John Barnard, Mike Blaskiewicz, Paul Channell, Alex Chao, Chipping Chen, Weiren Chou, Pat Colestock, Philip Efthimion, Nat Fisch, Alex Friedman, Irving Haber, Richard Hawryluk, Bill Herrmannsfeldt, Ingo Hofmann, Igor Kaganovich, Edward Lee, S. Y. Lee, Wei-li Lee, Grant Logan, Steve Lund, Robert Macek, Qian Qian, Martin Reiser, Gennady Shvets, Phillip Sprangle, Ed Startsev, Sean Strasburg, Stephan Tzenov, Han Uhm, Tai-Sen Wang, Tom Wangler, Jonathan Wurtele, Simon Yu, and Bruno Zotter.

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