PHYSICS OF NONNEUTRAL PLASMAS

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Published by

Imperial College Press
57 Shelton Street
Covent Garden
London WC2H 9HE

and

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN 1-86094-302-0

ISBN 1-86094-303-9 (pbk)

Printed in Singapore by Uto-Print

To Jean

*For as the sun is daily new and old,
So is my love still telling what is told.*

William Shakespeare

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Author's Foreword

Physics of Nonneutral Plasmas was first published in 1990 by Addison-Wesley (Reading, Massachusetts) when the author was Professor of Physics at the Massachusetts Institute of Technology. During the intervening period, research on nonneutral plasmas has evolved into a well-developed subfield of plasma physics, and I am delighted that World Scientific Publishing Co. has offered to reissue this graduate-level text on nonneutral plasmas, which has become a standard reference in this rapidly evolving area of physics research. *Physics of Nonneutral Plasmas* has been 'field-tested' in numerous graduate-level course offerings, including those given by the author at the Massachusetts Institute of Technology, Princeton University, and the United States Particle Accelerator School.

A nonneutral plasma is a many-body collection of charged particles in which there is not overall charge neutrality. The simplest examples are *one-component* pure ion plasmas, or pure electron plasmas. Since *Physics of Nonneutral Plasmas* was first published, this area of physics research has developed into a sophisticated subfield of pure and applied plasma physics. The diverse areas of application include: the nonlinear dynamics and collective processes in charge bunches in high-intensity accelerators for high energy and nuclear physics applications; investigations of nonlinear vortex dynamics and turbulence in nearly-inviscid two-dimensional fluid flow; the development of precision atomic clocks; coherent electromagnetic wave generation by energetic electrons interacting with magnetic field structures, as occurs in magnetrons, free electron lasers, and cyclotron masers; periodic focusing induction linac accelerators for the acceleration and transport of space-charge-dominated heavy ion beams;

research on properties of strongly correlated (including crystalline) one-component nonneutral plasmas; basic studies of the collective properties and nonlinear dynamics of laboratory-confined nonneutral plasmas in Malmberg-Penning traps and Paul trap configurations; studies of the basic thermal equilibrium and thermodynamic properties of one-component nonneutral plasmas; research on the formation and confinement properties of positron plasmas; investigations of the production of antihydrogen for basic physics studies by the mixing of positron and antiproton plasmas; and the equilibrium and stability of intense nonneutral electron and ion flow in high-voltage diodes, to mention a few examples. *Physics of Nonneutral Plasmas* provides an thorough physics foundation for advanced scientific research in these and related areas.

Physics of Nonneutral Plasmas has been prepared as a graduate-level text which covers a broad range of topics related to the fundamental properties and applications of nonneutral plasmas. The subject matter is treated systematically from first principles using a unified theoretical approach, and the emphasis is on the development of basic concepts that illustrate the underlying physical processes, which are often similar in different application areas. *Physics of Nonneutral Plasmas* includes 138 problems, 143 figures and illustrations, and the results from several classic experiments illustrating fundamental processes in nonneutral plasmas. In view of the book's emphasis on basic physics principles, and the thorough presentation format, it is intended to have a broad and lasting appeal to graduate students and researchers in the field.

Finally, because of the advanced theoretical techniques developed for describing the properties of one-component charged particle systems, *Physics of Nonneutral Plasmas* provides a useful companion volume to *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, Singapore, 2001) by Ronald C. Davidson and Hong Qin.

Ronald C. Davidson
Department of Astrophysical Sciences
Princeton University
October, 2001

Preface

A nonneutral plasma is a many-body collection of charged particles in which there is not overall charge neutrality. Such systems are characterized by intense self-electric fields, and in high-current configurations by intense self-magnetic fields. Nonneutral plasmas, like electrically neutral plasmas, exhibit a broad range of collective properties, such as plasma waves, instabilities, and Debye shielding. Moreover, the intense self fields in nonneutral plasmas can have a large influence on detailed plasma behavior, including stability and transport properties.

Since the monograph *Theory of Nonneutral Plasmas* (Benjamin, Reading, Massachusetts) first appeared in 1974, this important area of physics research has developed into a diverse and sophisticated subfield of pure and applied plasma physics. For example, interest in the physics of nonneutral plasmas has increased substantially in such diverse areas as: investigations of basic equilibrium, stability and transport properties; high-intensity accelerators for high energy and nuclear physics applications; phase transitions in strongly coupled, two- and three-dimensional nonneutral plasmas; coherent electromagnetic wave generation by free electrons interacting with applied magnetic field structures; astrophysical studies of large-scale isolated nonneutral plasma regions in the magnetospheres of rotating, magnetized neutron stars; and the development of positron and antiproton sources for antihydrogen production.

In addition to developing a basic physics understanding of many-body charged-particle systems in which there is not overall charge neutrality, there are many practical applications of nonneutral plasmas. These include: the development of precision atomic clocks;

investigations of nonlinear vortex dynamics and turbulence in nearly-inviscid two-dimensional fluid flow; coherent electromagnetic wave generation by energetic electrons, as occurs in magnetrons, free electron lasers, and cyclotron masers; the development of high-intensity accelerators, such as the periodic focusing induction linac accelerators for space-charge-dominated heavy ion beams; the stability of intense nonneutral electron and ion flow in high-voltage diodes; and the stability and transport of intense charged particle beams propagating through background plasma or through the atmosphere, to mention a few examples.

Physics of Nonneutral Plasmas has been prepared as a graduate-level text which covers a broad range of topics related to the fundamental properties and applications of nonneutral plasmas. The subject matter is treated systematically from first principles using a unified theoretical approach, and the emphasis is on the development of basic concepts that illustrate the underlying physical processes, which are often similar in different application areas. The statistical models used to describe the properties of nonneutral plasma are based on the fluid-Maxwell equations, the Vlasov-Maxwell equations, or the Klimontovich-Maxwell equations, as appropriate to the application under consideration.

Physics of Nonneutral Plasmas includes 138 problems, 143 figures and illustrations, and the results from several classic experiments illustrating fundamental processes in nonneutral plasmas. In view of the book's emphasis on basic physics principles, and the thorough presentation format, it is intended to have a broad and lasting appeal to graduate students and researchers in this rapidly developing field of physics research.

Ronald C. Davidson
Princeton, New Jersey
October, 2001

Acknowledgments

This book was first published in 1990 when the author was Professor of Physics at the Massachusetts of Technology. The material for this book evolved over two decades of teaching and research on the physics of nonneutral plasmas and intense charged particle beams. Many colleagues, students and collaborators, too numerous to mention, have contributed to virtually every topic covered in the book.

With regard to the selection of material incorporated in this graduate-level text, I am grateful to George Bekefi, Adam Drobot, David Hammer, Dennis Keefe, Martin Lampe, Tom O'Neil, Martin Reiser, Norman Rostoker, Richard Temkin, Han Uhm, David Wineland and Jonathan Wurtele for many valuable discussions and suggestions.

Preparing the manuscript was a significant undertaking. I am particularly indebted to Vera Sayzew for her skillful typing of the manuscript. I also wish to thank Albe Dawson for assistance in proof-reading, Steve Lund for preparing several of the graphical results, and Cathy Lydon for preparation of the figures.

In addition, I am very grateful to Brian Yang for his thorough technical review of the completed manuscript, including the model assumptions and results, equations, and problem sets. I also wish to thank David Chernin, Edward Lee, Martin Reiser and Lloyd Smith for their valuable comments regarding the material on alternating-gradient focusing systems included in Chapter 10.

The preparation of this book was possible only as a result of the encouragement of many individuals, institutions and government agencies. In this regard, I particularly wish to thank Robert Birge-neau, John Deutch and Ronald Parker of the Massachusetts Institute of Technology; Charles Roberson of the United States Office of Naval Research; Timothy Coffey and Sidney Ossakow of the United States Naval Research Laboratory; and David Sutter of the United States Department of Energy, High Energy Physics Division.

Finally, I would like to thank my wife Jean, daughter Cyndy, and son Ronald, Jr., for their strong support and encouragement during the preparation of the manuscript.

Ronald C. Davidson
Princeton, New Jersey
October, 2001

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