RESONANCE

Applications in Physical Science

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Applications in Physical Science

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Introduction

Many centuries ago, an intelligent individual could master a large proportion of the prevailing knowledge of the time and there were no accepted boundaries in science, as we know them today. Thus, Isaac Newton was not only a mathematician and physicist but also spent much of his later life in the practice of alchemy, a kind of chemistry not held in high regard, even in those days. Robert Hooke, a contemporary of Newton, had even wider interests, covering gravitation, astronomy, biology, palaeontology and the design of timepieces.

With the explosive expansion of scientific knowledge, particularly in the last few decades, it has become increasingly difficult to keep up-to-date even in one area of science — be it physics, chemistry, biology or any other well-defined subject area. Indeed, it is a well-rounded physicist who can understand even 50% of the titles in the publication *Physics Abstracts*, let alone the contents of the papers to which they refer.

It has been traditional to structure university science lecture courses in terms of mainstream topics (e.g. physical optics, structure and bonding, atomic physics, spectroscopy etc.) and to many students the links between different strands of their course never become clear. Not only do they not see the essential unity of science, but also some even lack a sense of unity in their own specialist area of study.

It is clearly impossible to return to the era of the universal scientist, but what *is* possible is to explore some topics which act like

weft in binding together different strands within a single subject area or even, to some extent, different subject areas. Various branches of mathematics serve this purpose: as an example, matrices can be used to analyse complex lens systems; to represent the Lorentz transformation in relativity theory; to express electron spin in quantum mechanics; and to study both vibrating systems and biological populations.

At the level of the trained scientist, the appreciation of these cross-links can give insights that lead to the better practice of science. X-ray crystallography is an example of a topic that binds together vast tracts of science such as mathematics, physics, chemistry, biology, geology and mineralogy — and there may be others. The American pure mathematician Herbert Hauptman (1917–2011), who became a theoretical crystallographer, eventually won a Nobel Prize in Chemistry!

Another of these linking topics is *resonance*, which occurs in various guises: in everyday life; in artistic contexts such as music; and in many forms in the physical sciences. The purpose of this textbook is to explore a variety of scientific areas in which resonance occurs, ranging from concerns about the safety of bridges to the confirmation of a prediction of Einstein's Theory of General Relativity.

Interspersed within the chapters are *exercises*, which are usually simple numerical applications of the immediately preceding material and are designed to instil a feeling for the magnitudes of the quantities of interest. At the end of chapters there are *problems*, which are more demanding than the exercises and whose purpose is to test the student's understanding of the material within each chapter. A solutions section provides a check on the student's success (or otherwise) in completing these tests, but where the student has difficulties, they also provide a guide that reinforces the teaching role of the chapter.

Before the advent of the computer age there were two branches of every science: *experimental* and *theoretical*. To these we must now add *computational*; there are many areas of science where both experimental and theoretical solutions to problems are impossible and the only solution is through computation. An outstanding example is the solution of many-body problems in astronomy;

there are some theoretical solutions for special three-body problems (see Section 4.4.4) but, in general, the motion of more than two bodies under their mutual gravitational forces requires a computational approach. Appendices II and III give programs to solve astronomical many-body problems, which are required for the complete solution of some of these problems.¹ They are in very basic FORTRAN77 code, but for those unfamiliar with FORTRAN the program will guide them in inserting the correct data, if they have access to a compiler.

A useful ability for most modern scientists is to be able quickly to write short programs to carry out straightforward calculations that would be tedious any other way. There are problems in this text where the value of a function of a single variable is required for a large number of values of the variable and simple programs consisting of few lines of code enable this to be done quickly. By outputting data to a file, a graphical solution can then be found with the use of a graphics package.

¹Copies of these programs are available for free download from:
<http://www.worldscientific.com/worldscibooks/10.1142/p963#t=suppl>