INVESTIGATING THE PEDAGOGY OF MATHEMATICS

How Do Teachers Develop Their Knowledge?

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How Do Teachers Develop Their Knowledge?

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Focusing on the Growth of Teachers' Knowledge: An Introduction

There is no doubt that, over the last few decades, general politicians, educational leaders, researchers, and practitioners have increasingly realized the crucial importance of teachers' knowledge in improving the quality of classroom teaching and learning. Accordingly, the issue of how teachers gain or develop their knowledge has also been a hot topic in educational research and on-going debate. Answering this issue calls for research-based evidences and sound knowledge for policy making and school practice in relation to teacher recruitment, teacher education, and teacher professional development.

This book presents two research studies focusing on how teachers develop their knowledge in the pedagogy of mathematics. The first study was conducted in the city of Chicago of the United States (hereafter called "the Chicago study") and the second one was conducted in Singapore (hereafter called "the Singapore study"). Both studies address the same research questions, but in different social, cultural, and educational contexts. The first question is, are there different sources for teachers to gain their pedagogical knowledge? And the second is, if the answer to the first question is "yes", then how do different sources contribute to the development of teachers' pedagogical knowledge?

The book consists of two main parts. Part I is devoted to the Chicago study in a relatively detailed way. It contains the following nine chapters.

Chapter 1 introduces the background, the significance, and the research questions of the study.

Chapter 2 first presents some epistemological background about the concept of knowledge for the study, and then provides a relatively broad review of the literature in the field of teacher knowledge, which is divided around three major issues: what knowledge do teachers need, what knowledge do teachers have, and how do teachers develop their knowledge.

Chapter 3 establishes a conceptual framework to examine teachers' pedagogical knowledge and the possible different sources from which teachers develop their pedagogical knowledge.

Under the conceptual framework, teachers' pedagogical knowledge is further classified into three components, namely, pedagogical curricular knowledge (PCrK): knowledge of teaching materials and resources, including technology; pedagogical content knowledge (PCnK): knowledge of ways to represent mathematics concepts and procedures; and pedagogical instructional knowledge (PIK): knowledge of general teaching strategies and classroom organizational models.

Chapter 4 introduces the research design, instruments, and procedures used for collecting and analyzing the data, which is gathered through a questionnaire survey, classroom observation, and interviews from 77 mathematics teachers in three high-performing high schools, a stratified random sample from the 25 best high schools in the metropolitan area of Chicago.

Chapters 5, 6, and 7 report the core findings of the Chicago study. More specifically, Chapter 5 reports the results about how teachers develop their PCrK, Chapter 6 is about how teachers develop their PCnK, and Chapter 7 is about how teachers develop their PIK.

Chapter 8 reports other findings of the study, supplementing the three earlier chapters by looking at some other issues concerning the development of teachers' pedagogical knowledge. It looks at each specific source identified in the study and does not separate the general pedagogical knowledge into three different components.

Chapter 9 provides a summary of the Chicago study and its main conclusions. It also discusses the implications of the findings for teacher educators, school administrators, and teachers themselves on how to effectively pursue the growth of teachers' pedagogical knowledge. The chapter also offers suggestions for further research concerning the development of teacher knowledge.

Part II of the book comprises Chapter 10 and Chapter 11. Chapter 10 presents the Singapore study, but in a more concise manner, given the fact that it is largely a replication of the Chicago study in terms of conceptualization and research methods. Accordingly, more emphasis of the writing is placed on the Singapore study's different social, cultural, and educational context compared with the Chicago study's. The Singapore study has a purpose of confirmation and comparison of the findings with the Chicago study. The data was collected from 73 mathematics teachers in six secondary schools, a stratified random sample from all of the 152 secondary schools in Singapore.

Chapter 11 juxtaposes the Singapore study with the Chicago one and compares the similarities and differences of the results as revealed in the two studies. Some concluding remarks are offered at the end of the chapter.

Both the Chicago and Singapore studies revealed that there are various sources from which teachers can develop their pedagogical knowledge. In particular, both studies consistently found that teachers' own teaching experience and reflection is the most important source for them to develop all the three components of pedagogical knowledge, and teachers' daily or informal exchanges with their colleagues is the second most important source for all the three components of their pedagogical knowledge. For the other sources, the studies revealed, to a greater or lesser degree, there exists different importance in teachers' developing one or more components of their pedagogical knowledge. In addition, the teachers in Chicago reported that preservice training was the least important source, while their Singapore counterparts viewed it as a secondarily important source, for the development of their pedagogical knowledge.

It should be pointed out that the Chicago study was originally completed as the author's doctoral thesis and, after translation into Chinese, first published in a book series by East China Normal University Press, Shanghai, China in 2003. As a researcher and the author, I was heartened to know that the book has had a positive impact on China's national policy about school-based teacher professional development,¹ and was regarded

¹ See Jian Liu's comments on the Chinese edition of the Chicago study. *A Study on the Development of Teachers' Pedagogical Knowledge* (2nd edition, back cover), Shanghai: East China Normal University Press, 2013.

by researchers, generously, as one of the most influential studies in this area,² being widely used as a university graduate course book. According to the information provided by Dr. Dongchen Zhao, it has been cited 833 times in China (source: www.cnki.net, as of 1 March 2014) including 100 doctoral theses but excluding citations in books. In consideration of the demand from the readers, the publisher has decided to publish the second edition of the book in Chinese. I am delighted that the English version of the Chicago study, of which much original integrity is kept with some necessary changes, together with a follow-up study, the Singapore study, is now published by Imperial College Press.

Although the studies presented in this book were conducted in Chicago and Singapore with focus on pedagogical knowledge of mathematics teachers, I hope that the book can be of value not only to readers in these two countries and mathematics educators who will naturally feel it easier to understand the relevant background of the studies, but also to any educational policy makers and researchers, teacher educators, school administrators and teachers, and graduate students who are interested in the theme of teachers' knowledge development. The reason is that the issue of how teachers gain their knowledge has general importance and communality in different countries and across different subject matters, and it is also internationally a common challenge to continuously improve teachers' professional knowledge.

Many people have offered me a lot of important help in conducting the Chicago and Singapore studies and in writing and publishing this book. For this, I am indebted to my professors, colleagues, research assistants, and friends, in particular, Zalman Usiskin, Robert Dreeben, John Craig, Kenneth Wong, Susan Stodolsky, John McConnell, Steve Victora, Lilane Koehn, Timothy Kanold, Kwai Ming Wa, David Witonsky, Alfred Estberg, Suzanne Levin, Eileen Fernández, Susan Chang, and Tad Waddington for the Chicago study, and Christina Cheong, Lee Peng Yee, Lionel Pereira-Mendoza, Kho Tek Hong, Cheong Peck Yoke, Teo Yin Nah, Yeo Jia Chyang, Yeo Jinguang Mathew, and Zhu Yan for the Singapore study.

²For example, see Zhang (2009).

I am also indebted to all the school teachers, heads of mathematics departments, and administrators in Chicago and Singapore who participated or helped in various ways in the two studies, though because of the agreement for conducting the studies, their names cannot be listed in this book.

I must thank my wife Ellen and our daughters Jenny and Kate for their patience and sacrifice for supporting me to write this book. In addition, I wish to thank Alice Oven and Tasha D'Cruz of Imperial College Press for their highly professional service and help in publishing the book. I also wish to thank my colleagues at Southampton: Keith Jones, Jacky Lumby and Daniel Muijs for their helpful suggestions for this book.

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