The Theory of
TOROIDALLY CONFINED PLASMAS
Third Edition

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TOROIDALLY CONFINED PLASMAS

Third Edition

Roscoe B. White

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兵の法を學ばんと思はば、此書を思案して、師
は針弟子は糸と成て、絶えず稽古あるべき事也。

*"If you want to learn the art
search beneath the surface.
The teacher is as a needle,
the student is as thread. You
must practice constantly."*

A Book of Five Rings
Miyamoto Musashi, 1645

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Preface to the Third Edition

Since publishing the second edition of this book, I have been very much involved with examining the nature of mode particle interactions in toroidal confinement devices. This work has resulted in a large extension of the chapter on the theory of mode-particle interactions. There is also a full chapter devoted to the use of lithium as a particle sink either on the plasma wall or on the divertor. If the use of lithium turns out to be worth even a small fraction of its theoretically possible utility, it will greatly help the program. The development of a liquid lithium divertor is presently being carried out in China. Research on its use is also being carried out at Princeton, Frascati, and Padova.

Recent good news is the confirmation of the plans to proceed with the construction of IGNITOR, partly constructed in Italy and planned to be built at the trinity site at Troitsk, near Moscow. An important missing part of the experimental data to date is information obtained from the observation of an ignited burning plasma, or at least one in which the fusion energy is an important part of the plasma heating.

A section on nonlocal transport has been added, since it has increasingly become apparent that simple local descriptions of transport using locally derived diffusion operators are not sufficient to describe transport in toroidal devices. This is particularly apparent in the reversed field pinch at Padova, since many discharges consist of magnetic field structures that are above the stochastic threshold.

There is as well a new chapter deriving equations for full cyclotron motion in a toroidal system and the comparison of some results with guiding center simulations. In some present devices, in particular in NSTX, the

cyclotron radius is very large, leading to some question concerning the use of guiding center equations to describe particle motion.

The presentation is necessarily not uniform; the sections treating the derivation of basic magnetohydrodynamic equilibrium and stability properties, and of particle orbits, are more mathematical, whereas other sections are more approximate and descriptive in nature. This is to provide a basic foundation and at the same time arrive at a description of current phenomena without becoming overwhelmed by minute detail.

Roscoe White
Princeton Plasma Physics Lab
Princeton University, 2013

Preface to the Second Edition

In the intervening years since the first edition of this book was published the field of magnetically confined fusion research has made great advances, and the goal of an inexpensive, abundant, clean, and inexhaustable energy source is tantalizingly close. The record value of fusion power produced in research reactors has increased even faster than the memory capacity of semiconductor chips, doubling approximately every year. The large tokamak at Princeton, TFTR, reached fusion power output greater than one fourth of the energy input, producing 11 Megawatts of fusion power, and the joint European tokamak in England, JET, passed the 60 percent mark, producing 16 megawatts of fusion power. Both JET and the Japanese reactor JT-60, operating without tritium, approached the equivalent break even point, *i.e.* in the same discharge; with tritium present, they would have produced as much power as used to heat the discharge. Research tokamaks regularly produce temperatures ten times hotter than the interior of the sun, and sustained stable operation at lower temperatures has been maintained for over an hour. In specially configured discharges in DIII-D at General Atomics the transport was reduced to the theoretical minimum neoclassical limit by using sheared rotation to stabilize the turbulence. Most of the rapid large scale instabilities impeding confinement have been understood and dominated, and research focuses on improving and extending the operating domain by the control of fine-scale instabilities.

At the same time the field experienced an often rancorous debate concerning the wisdom of constructing a large multibillion dollar demonstration reactor capable of igniting, producing net power, and testing associated engineering components. The reactor, ITER, was designed as a joint effort by

the four partners, Europe, the Soviet Union, Japan, and the United States. The debate was complicated by the collapse of the Soviet Union and the withdrawal of the United States from all but token theoretical analysis due to congressional pressure to balance the US budget. ITER is now being built in Cadarache, France, and China, India and South Korea are also participating. In addition, large independent programs are underway in Japan, China and South Korea.

Many US physicists feel that a reactor produced following the design of ITER cannot be economically successful, and the US program, aside from an official but small support for ITER, has retreated to a search for a “smaller, cheaper, smarter” solution. One such approach was guided by the discovery of a large class of stellarators, the so called quasi-symmetric stellarators, which could offer distinct advantages over tokamaks provided they succeed in obtaining high beta and reasonable confinement. The parameter space of aspect ratio, elongation, and cross-sectional shape is being explored, as is the use of various means of profile control to produce regimes of very good confinement. The idea of covering the plasma-facing toroidal walls with flowing liquid metal, to eliminate the problems associated with wall erosion and the design of divertors. An important missing experimental step is the production of a burning, ignited plasma, and most physicists agree that such an experiment is crucial. Aside from ITER, the only design for an ignited plasma device to proceed beyond paper studies is the IGNITOR, partly constructed in Italy, but its future completion is uncertain.

In spite of the discord concerning the research strategy, the field is rather optimistic. The impressive list of successes obtained in the last decades leads many research fusion scientists to believe that a good solution is “out there” waiting to be found, and that it will be found. In the meantime, relatively cheap petroleum and natural gas, along with the development of renewable energy sources continue to suppress the urgency for arriving quickly at an economical design. The minimum commitment allowing preservation of the results established so far is to continue to train and maintain a cadre of professionals capable of carrying out the search, and of rapidly producing a good reactor when such a design is found.

The book has been extended in several ways. Many of the derivations have been improved. Wherever possible the analysis has been extended to include other toroidal devices as well as tokamaks, including the analysis of equilibrium, particle orbits and stability. The area of field stochasticity and the onset of chaos has been improved. The treatments of bootstrap current,

stochastic ripple loss, and phase integral methods have all been rewritten and extended, and new sections added treating the classification of particle orbits in tokamaks, TAE modes and their saturation, neoclassical tearing, and general wave-particle interactions.

The second edition has benefited from comments and corrections by Mike Beer, Alain Brizard, Yanlin Wu, Wonchull Park, Don Monticello, Allan Reiman, Wei Li Lee, Zhihong Lin, Rob Goldston, Paul Parks, Stuart Zweben, Leonid Zakharov, and many others, as well as the availability of LATEX word processing. Many of my students have contributed by posing good questions and by showing interest in the further development of some topics.

Roscoe White
Princeton Plasma Physics Lab
Princeton University, 2001

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Preface to the First Edition

These notes accompany a graduate course taught at Princeton, designed to provide a basic introduction to plasma equilibrium, particle orbits, transport, and those ideal and resistive magnetohydrodynamic instabilities which dominate the behavior of a tokamak discharge, and to develop the mathematical methods necessary for their theoretical analysis. Primarily the book covers the consequences of ideal and resistive magnetohydrodynamics, these theories being responsible for most of what is well understood regarding the physics of tokamak discharges. No attempt is made to discuss the derivation of this formalism, this being a topic better left to a course devoted to kinetic theory. The focus is rather on the description of equilibria, the linear and nonlinear theory of large scale modes, and single particle guiding center motion, including simple neoclassical effects. Modern methods of general magnetic coordinates are used, and the student is introduced to the onset of chaos in Hamiltonian systems in the discussion of destruction of magnetic surfaces. The Hamiltonian formulation of guiding center motion provides another glimpse of modern methods in particle dynamics, preparing the way for the study of gyrokinetics. The interaction of a high energy particle component with a background magnetohydrodynamic plasma, of interest for the description of intensely heated and ignited plasmas, is also treated.

Much of the book is devoted to the description of the limitations placed on operating parameters given by ideal and resistive modes, and current ideas about how to extend and optimize these parameters. This permits the student to quickly arrive at the research level of a topic which is reasonably well developed and plays an important role in our current understanding of

the dynamics of basic confinement behavior. The part of the book dealing with transport consists of an elementary introduction to the principle neo-classical mechanisms, and examples of the perturbation methods employed to deduce transport rates from the drift kinetic equation. There is a brief introduction to some of the primitive theories and phenomenological descriptions of anomalous transport. Particle loss due to symmetry breaking perturbations such as toroidal field ripple is treated. The last chapter is a treatment of the method of phase integrals, which has proved very valuable in the study of ballooning modes, parametric instabilities, and microinstabilities. In spite of its elegance and utility there is no treatment in the literature easily accessible to the student.

To a large degree the book is self contained, and most chapters depend logically on material developed previously. The exceptions are the chapter on phase integral methods, which is independent, and the chapter on transport, which requires only Chap. 1 through 3. The course is aimed at second year graduate students, but by changing emphasis on the subjects presented, it could also be given to first year students, or even to undergraduates with some previous plasma physics training. At the same time, the material in each topic can be easily expanded to make contact with current research, and the course could be taught at a more advanced level by using the text as a base. Notation is uniform and corresponds to that most commonly used among working plasma theorists.

Roscoe B. White
Princeton 1989

Acknowledgement

The material for these lectures has come into being during interaction with many colleagues, both at Princeton and elsewhere. The subject matter necessarily reflects my interests and the interests of the people with whom I have worked. Particularly important were Paul Rutherford, who suggested the original outline of topics when I first taught the course and provided notes for some sections, Harry Mynick and Allen Boozer in the fields of guiding center motion and magnetic coordinates, Morrell Chance and Russell Kulsrud for magnetic coordinates and ideal MHD, Liu Chen, Jim Strachan, and Rob Goldston on the fishbone mode, and Marshall Rosenbluth, Don Monticello, Wonchull Park, and Kevin McGuire on resistive instabilities. I am indebted to Francis Troyon, James Van Dam, and Burton Fried for valuable suggestions regarding the text.

The second and third editions benefited from comments and corrections by Mike Beer, Alain Brizard, Yanlin Wu, Wonchull Park, Don Monticello, Allan Reiman, Wei Li Lee, Zhihong Lin, Rob Goldston, Paul Parks, Stuart Zweben, Leonid Zakharov, and many others, as well as by the availability of Latex word processing. The third edition has also benefited from my long-time interaction with Gianluca Spizzo and other physicists at the Reversed Field Pinch experiment in Padova, and close collaboration with Leonid Zakharov, who has carried out most of the theoretical analysis of the effect of lithium-coated walls and divertors, and many discussions with Stuart Hudson. Many of my students and also students of Zhihong Lin at UC Irvine have contributed by posing good questions and by showing interest in the further development of some topics.

Thanks are due to Barbara Sarfaty and Terry Greenberg for their care and perseverance in typing the first draft of the text, and finally much gratitude to Laura and Veronica for their patience while the book was being written.

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